Practice Midterm 2 – Math 2255

** **Practice midterm I** provides good practice problems for previous material.
** **Bring a single double-sided** 8.5×11 **sheet of notes to use during the final**.

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (2 points) If $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$ then $\mathcal{L}\{f(t)g(t)\} = F(s)G(s)$

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a piecewise continuous function on the interval $[0, 5]$ which is not continuous on the interval $[0, 5]$.

3. (10 points) Using **only the definition of the Laplace transform** compute the Laplace transform $\mathcal{L}\{u_2(t)t\}$.

4. (10 points) Find the inverse Laplace transform $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-3)} + \frac{1}{(s-4)^3}\right\}$.

5. (10 points) Express the function
$$f(t) = \begin{cases} 3, & 0 \le t < 2 \\ 6, & 2 \le t < 3 \\ -5, & 3 \le t \end{cases}$$
as a linear combination of Heavyside functions $u_c(t)$.

6. (10 points) Express the function
$$g(t) = \begin{cases} \sin t, & 0 \le t < 4 \\ t^2, & 4 \le t < 7 \\ 8t, & 7 \le t \end{cases}$$
as sum of products of Heavyside functions $u_c(t)$ with continuous functions.

7. Find the general solution to
$$t^2y'' - 2y = 3t^2 - 1$$
for $t > 0$ given that the general solution to the homogeneous equation $t^2y'' - 2y = 0$ for $t > 0$ is
$$c_1t^2 + c_2t^{-1}.$$

8. (10 points) Compute the convolution $f * g$ if
$$f(t) = \begin{cases} 0, & t \le -1 \\ 1 - t^2, & -1 \le t \le 1 \\ 0, & 1 \le t \end{cases}$$
and $g(t) = t$.

9. (10 points) Find the general solution to
$$y'' + 3y' - 2y = 6e^{2t}.$$

10. (10 points) Find the general solution to
$$y^{(6)} + 9y^{(4)} + 27y'' + 27y = 0.$$

11. (10 points) Solve the initial value problem

$$y'' + 4y = g(t)$$

where

$$g(t) = \begin{cases} 0, & 0 \le t < 2 \\ 1, & 2 \le t \end{cases}$$

and

$$y(0) = 0, \qquad y'(0) = 0.$$

12. (10 points) Solve the initial value problem

$$y'' + 4y' + 4y = \delta(t-3)$$

where δ is the Dirac delta function and

$$y(0) = 1, \qquad y'(0) = 0.$$