Practice Final – Math 2415

Practice midterms I and II provide good practice problems for previous material.

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (2 points) An $n \times n$ matrix has 0 as an eigenvalue if and only if it has determinant 0

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a Hermitian matrix which is not symmetric.
   (b) (2 points) Give an example of a 3×3 matrix with eigenvalues 2 and -7.
   (c) (2 points) Give an example a first order linear system of differential equations which is not homogeneous.

3. (5 points) Compute the Wronskian $W[\mathbf{u}, \mathbf{v}](t)$ for the vector valued functions

$$\mathbf{u}(t) = \begin{pmatrix} t^2 \\ t^3 \end{pmatrix}, \qquad \mathbf{v}(t) = \begin{pmatrix} t-1 \\ e^t \end{pmatrix}$$

4. (10 points) Find the general solution for the following linear system of differential equations

$$\begin{aligned} x_1'(t) &= x_1(t) - x_2(t) \\ x_2'(t) &= 2x_1(t) - 6x_2(t). \end{aligned}$$

   Sketch the phase portrait for these solutions.

5. (10 points) Find the solution for the following linear system of differential equations

$$\begin{aligned} x_1'(t) &= 3x_1(t) + 5x_2(t) \\ x_2'(t) &= 4x_1(t) + 4x_2(t). \end{aligned}$$

   which satisfies $x_1(0) = 1$ and $x_2(0) = 0$.

6. (10 points) Find the general solution for the following linear system of differential equations

$$\begin{aligned} x_1'(t) &= x_1(t) - 5x_2(t) \\ x_2'(t) &= 2x_1(t) - x_2(t). \end{aligned}$$

   Prove that you have a general solution using the Wronskian.

7. (10 points) Find the solution to the heat equation

$$4u_{xx} = u_t, \qquad 0 < x < 5, \qquad t > 0.$$

   which satisfies the boundary conditions

$$u(0,t) = 0, \qquad u(5,t) = 0, \qquad t > 0.$$

   and

$$u(x,0) = \sin(2\pi x/5) + 4\sin(2\pi x)$$

8. (10 points) Find the solution to the wave equation

$$4u_{xx} = u_{tt}, \qquad 0 < x < 5, \qquad t > 0$$

which satisfies the boundary conditions

$$u(0,t) = 0, \qquad u(5,t) = 0, \qquad t > 0,$$

$$u(x,0) = 0, \qquad 0 < x < 5,$$

and

$$u_t(x,0) = \sin(2\pi x/5) + 4\sin(2\pi x), \qquad 0 < x < 5.$$

9. (10 points) Use separation of variables to rewrite the partial differential equation

$$u_{xx} + u_x + u_{tt} = 0$$

as two ordinary differential equations.