Practice Midterm 1 – Math 2415

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (1 point) If the functions φ and ψ are solutions to the differential equation
   $$y'' + ay = 0$$
   then $\varphi - 2\psi$ is a solution to the differential equation as well.
   (b) (1 point) If either of the functions p or g is not differentiable at $t = t_0$ then the initial value problem
   $$y' + p(t)y = g(t), \qquad y(t_0) = y_0$$
   cannot have a unique solution on any open interval I containing t_0.
   (c) (1 point) It is possible for $\varphi(t) = e^t$ and $\psi(t) = 1$ to be solutions to the same first order differential equation
   $$y' = f(y, t)$$
   where f and $\frac{\partial f}{\partial y}$ are continuous on the entire (y, t)-plane.
   (d) (1 point) Let p and q be continuous on $\mathbf{R}$. Let y_1 and y_2 be solutions to the differential equation
   $$y'' + p(t)y' + q(t)y = 0.$$
   If the Wronskian determinant $W(y_1, y_2)(t_0)$ is 0 at t_0 then
   $$W(y_1, y_2)(t) = 0$$
   for all $t \in \mathbf{R}$.
2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a first order differential equation which is separable but not linear.
   (b) (2 points) Give an example of a nonhomogeneous 3rd order linear differential equation with **constant** coefficients.
   (c) (2 points) Give an example of a first order autonomous differential equation with exactly two equilibrium solutions.
3. (5 points) Compute the Wronskian for $y_1(t) = t^2$, $y_2(t) = \frac{1}{t}$
4. (5 points) Find the integrating factor for the first order linear equation
   $$ty' - 6y - t^2 + 1 = 0.$$
5. Find general solutions for the following differential equations
   (a) (5 points) $y' = (1 + y^2)e^x$.
   (b) (5 points) $y'' - 4y' + 4y = 0$.
   (c) (5 points) $y'' - 5y' - 6y = 0$.
   (d) (5 points) $y'' - 4y' + 13y = 0$.
   (e) (5 points) $y' - t^3y + 1 = 0$.
6. Find solutions for the following initial value problems
   (a) (5 points) $y'' + 3y' = 0, \quad y(0) = 3, \quad y'(0) = 1.$
   (b) (5 points) $y'' + 6y' + 10 = 0, \quad y(0) = 1, \quad y'(0) = 0.$
   (c) (5 points) $ty' + y = 6, \quad y(1) = 3.$
   (d) (5 points) $y'' + 8y' + 20y = 0, \quad y(0) = 6, \quad y'(0) = -1.$

7. (10 points) Sketch solutions to the equation

$$y' = \ln |y|.$$

on the (y, t)-plane. Be sure to include all equilibrium solutions and note whether they are stable, unstable or semistable. Also include a nonequilibrium solution above and below each equilibrium solution.

8. (10 points) t^2 is a solution to the differential equation

$$3t^2y'' - 2ty' - 2y = 0.$$

Find the general solution.

9. (10 points) A forest has the capacity to support 1000 elk but the elk will die off if fewer than 20 are present. The initial population is 100 and at that population size the growth rate is 20% per year. Give a model for logistic growth with a threshold for this population. Sketch the solution to this initial value problem.

10. (10 points) Suppose that the force of drag on a ball of mass m falling through the air were proportional to the velocity v of the ball. Set up a model for the velocity of the ball falling through the air if the acceleration of gravity is g. Find the velocity as a function of time if the initial velocity is 0.

11. (10 points) Show that the functions $y_1(t) = t$ and $y_2(t) = \sin(t)$ cannot both be solutions to a single differential equation of the form

$$y'' + p(t)y' + q(t)y = 0$$

where p and q are continuous on $\mathbf{R}$.