Practice Midterm 2 – Math 2415

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (1 point) Any periodic function with period T can be written in the form
   $$f(t) = R\cos\left(\frac{2\pi t}{T} + \delta\right)$$
   for some constants R and δ.
2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a continuous periodic function with fundamental period 2.
3. (5 points) Find the smallest vector space of functions closed under differentiation which contains the function
   $$f(t) = t^2 \sin 2t.$$
   You do NOT need to justify your answer.
4. (5 points) Find the smallest vector space of functions closed under the linear operator t^2D^2 which contains the function
   $$f(t) = t^4$$
   You do NOT need to justify your answer.
5. (5 points) Find the values R and δ for which
   $$\cos t - 2\sin t = R\cos(t - \delta)$$
   for all $t \in \mathbf{R}$.
6. Find general solutions for the following differential equations
   (a) (5 points) $y'' - 4y' + 5y = t^2$.
   (b) (5 points) $y'' - 4y' + 4y = te^{2t}$.
   (c) (5 points) $y'' - 5y' - 6y = 3t - e^{-t}$.
   (d) (5 points) $y'' - 4y' + 13y = e^t \cos t$.
7. Find solutions for the following initial or boundary value problems
   (a) (5 points) $y'' + 3y' = e^{-3t}, \quad y(0) = 0, \quad y'(0) = 1.$
   (b) (5 points) $y'' - y' - 5y = 0, \quad y'(0) = 0, \quad y(1) = e.$
   (c) (5 points) $y'' + 7y = 6, \quad y(0) = 0, \quad y(1) = 3.$
8. (10 points) Find the eigenfunctions with real eigenvalues for the differential operator D^2 which satisfy the boundary conditions $y(0) = 0$ and $y(2\pi) = 0$.
9. (10 points) When a $10\,\mathrm{kg}$ mass is attached to a spring the spring stretches $0.5\,\mathrm{m}$. Find the natural angular frequency ω_0 of the spring if there is no damping. Express your answer as a function of acceleration g. (Do not substitute $9.8\,\mathrm{m/s^2}$ in for g)
10. (10 points) Find the Fourier Series for the function f with period 4 satisfying
    $$f(x) = \begin{cases} 0, & -2 \le x < 0 \\ x - 1, & 0 \le x < 2 \end{cases}$$

11. (10 points) An external force of the form $F(t) = F_0 \cos \omega t$ is applied to a $2\,\mathrm{kg}$ mass attached to a spring with spring constant $k = 8\,\mathrm{N/m}$ where $F_0 = 5\,\mathrm{N}$. The system has a damping constant of $\gamma = 2\,\mathrm{Ns/m}$. Find the frequency ω at which the system will exhibit resonance. What amplitude do you expect for the motion at that resonant frequency.

12. (10 points) Show that if the functions f and g are periodic with fundamental period T then $f - g$ is periodic with period T. Must $f - g$ have fundamental period T?