Math 2415 - Additional Practice for Final

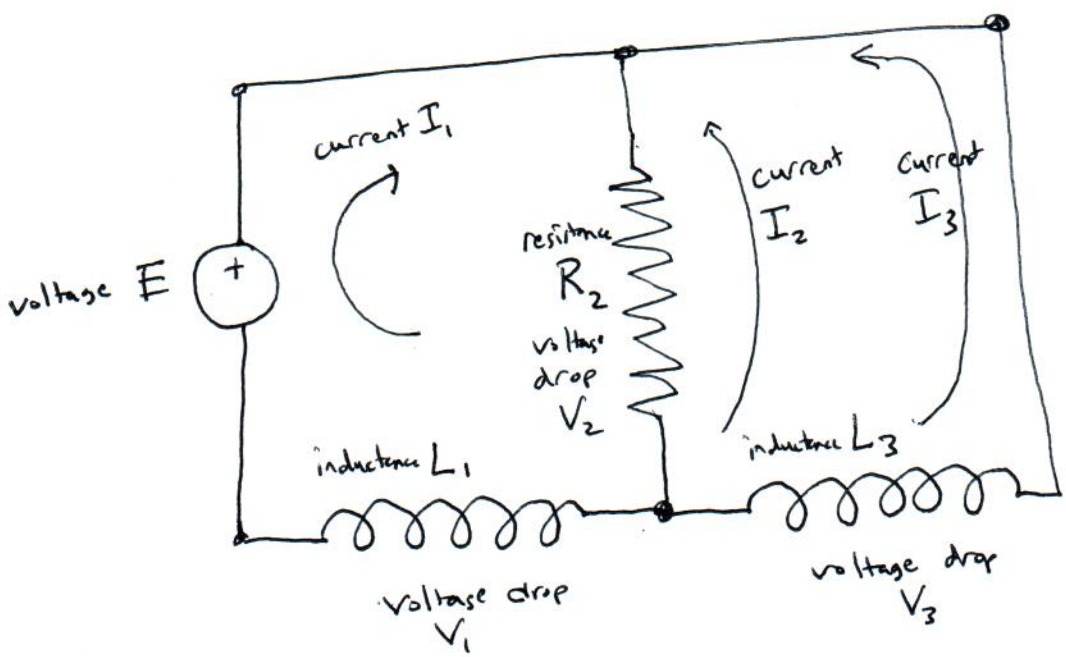
(10) For the circut diagram pictured Give a system of first-order linear equations satisfied by the vector-valued function

$$\vec{x}(t) = \begin{pmatrix} I_1(t) \\ I_3(t) \end{pmatrix}$$

of the form

$$\vec{x}' = A\vec{x} + \vec{g}(t)$$

where A is a constant matrix.



(10) Solution

Kirchoff's Rules give

$$I_1 + I_2 + I_3 \overset{(i)}{=} 0$$

$$V_1 - E \overset{(ii)}{=} V_2 \overset{(iii)}{=} V_3$$

voltage drops give the equations:

$$V_1 \overset{(iv)}{=} L_1 I_1'$$

$$V_2 \overset{(v)}{=} R_2 I_2$$

$$V_3 \overset{(vi)}{=} L_3 I_3'$$

Using (i) eliminate I_2

$$I_2 = -I_1 - I_3$$

Using (v) eliminate V_2

$$V_2 = R_2 I_2 = R_2 (-I_1 - I_3) = -R_2 I_1 - R_2 I_3$$

Using (ii) eliminate V_1

$$V_1 = V_2 + E = -R_2 I_1 - R_2 I_3 + E$$

Using (iii) eliminate V_3

$$V_3 = V_2 = -R_2 I_1 - R_2 I_3$$

(iv) becomes

$$-R_2 I_1 - R_2 I_3 + E = L_1 I_1'$$

(vi) becomes

$$-R_2 I_1 - R_2 I_3 = L_3 I_3'$$

This gives the first order system

$$I_1' = -\frac{R_2}{L_1} I_1 - \frac{R_2}{L_1} I_3 + \frac{E}{L_1}$$

$$I_3' = -\frac{R_2}{L_3} I_1 - \frac{R_2}{L_3} I_3$$

or

$$\vec{X}' = \begin{pmatrix} -\frac{R_2}{L_1} & -\frac{R_2}{L_1} \\ -\frac{R_2}{L_3} & -\frac{R_2}{L_3} \end{pmatrix} \vec{X} + \begin{pmatrix} \frac{E}{L_1} \\ 0 \end{pmatrix}$$

where $\vec{X} = \begin{pmatrix} I_1 \\ I_3 \end{pmatrix}$