Important definitions

Definition (pg. 38). $n$ is an **even number** if there is $k \in \mathbb{Z}$ such that $n = 2k$.

Definition (pg. 38). $n$ is an **odd number** if there is $k \in \mathbb{Z}$ such that $n = 2k + 1$.

Definition (pg. 41). $q$ is a **rational number** if there is $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ such that $b \neq 0$ and $q = \frac{a}{b}$.

Definition (pg. 42). $q$ is an **irrational number** if $x \in \mathbb{R}$ and $x$ is not rational.

Definition (pg. 43). Let $d, x \in \mathbb{Z}$. $d$ divides $x$ (or $x$ is divisible by $d$) if there is $k \in \mathbb{Z}$ such that $x = dk$.

Definition (pg. 45). Let $x \in \mathbb{N}$. $x$ is a **prime number** if $x \neq 1$ and for all $a, b \in \mathbb{N}$ if $x = ab$ then $a = 1$ or $b = 1$.

Definition (pg. 48). Let $a, b, m \in \mathbb{Z}$. $a$ is congruent to $b$ modulo $m$ ($a \equiv b \pmod{m}$) if $m$ divides $b - a$.

Definition (pg. 58). For all $n, k \in \omega$ then the **binomial coefficients** $\binom{n}{k}$ satisfy the following two rules:

1. For all $n \in \omega$ we have $\binom{n}{0} = \binom{n}{n} = 1$.
2. For all $n, k \in \mathbb{N}$ if $k \leq n$ then $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Definition (pg. 99). The **empty set** $\emptyset$ is the set such that for all $x$ we have $x \notin \emptyset$.

Definition (Lecture 22). Let $A$ and $B$ be sets. $A$ equals $B$ ($A = B$) if for all $x$ we have $x \in A$ if and only if $x \in B$.

Definition (pg. 100). Let $A$ and $B$ be sets. $A$ is a **subset** of $B$ ($A \subseteq B$) if for all $x \in A$ then $x \in B$.

Definition (pg. 100). Let $A$ and $B$ be sets. $A$ is a **proper subset** of $B$ ($A \subset B$) if $A \subseteq B$ and $A \neq B$.

Definition (pg. 101). Let $A$ and $B$ be sets. The **union** of the sets $A$ and $B$ is the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$ 

Definition (pg. 101). Let $A$ and $B$ be sets. The **intersection** of the sets $A$ and $B$ is the set

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$ 

Definition (pg. 101). Let $A$ and $S$ be sets. The **relative complement** of the set $A$ in the set $S$ is the set

$$S \setminus A = \{x \mid x \in S \text{ and } x \notin A\}.$$ 

Definition (pg. 105). Let $A$ and $B$ be sets. The sets $A$ and $B$ are **disjoint** if $A \cap B = \emptyset$.

Definition (pg. 108). Let $\mathcal{A}$ be a set of sets. The **union** of $\mathcal{A}$ is the set

$$\bigcup \mathcal{A} = \{x \mid \text{There exists } A \in \mathcal{A} \text{ such that } x \in A\}.$$ 

Definition (pg. 108). Let $\mathcal{A}$ be a nonempty set of sets. The **intersection** of $\mathcal{A}$ is the set

$$\bigcap \mathcal{A} = \{x \mid \text{For all } A \in \mathcal{A} \text{ such that } x \in A\}.$$ 

Definition (pg. 109). Let $A$ be a set. The **power set** of $A$ is the set

$$\mathcal{P}(A) = \{S \mid S \subseteq A\}.$$ 

Definition (pg. 110). **Ordered pairs** $(a, b)$ and $(c, d)$ are equal if and only if $a = c$ and $b = d$. 
Definition (pg. 112). Let $A$ and $B$ be sets. The **Cartesian product** of $A$ and $B$ is the set

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$  

Definition (pg. 126). Let $A$ and $B$ be sets. A **function** $f$ from $A$ to $B$ is a subset $f \subset A \times B$ satisfying that for all $a \in A$ there is a unique $b \in B$ such that $(a, b) \in f$ (we usually write $f(a) = b$ instead of $(a, b) \in f$).

Definition (pg. 113). Let $A$ and $B$ be sets. If $f : A \to B$ then the **domain** of $f$ is the set $\text{Dom}(f) = A$.

Definition (pg. 114). Let $A$ be a set. The **identity function** on $A$ is the function $\text{id}_A : A \to A$ satisfying that for all $a \in A$ we have $\text{id}_A(a) = a$.

Definition (pg. 114). Let $A \subset B$. The **inclusion function** from $A$ into $B$ is the function $i : A \to B$ satisfying that for all $a \in A$ we have $i(a) = a$.

Definition (pg. 114). Let $A$ be a set. The **empty function** $\emptyset$ into $A$ is the unique function satisfying $\emptyset : \emptyset \to A$.

Definition (pg. 115). Let $A$ and $B$ be a sets. The **projection functions** are the functions $\pi_A$ and $\pi_B$ satisfying $\pi_A : A \times B \to A$ and $\pi_B : A \times B \to B$ and for all $a \in A$ and $b \in B$ we have $\pi_A(a, b) = a$ and $\pi_B(a, b) = b$.

Definition (pg. 117). Let $f$ be a function. The **range** of $f$ is the set

$$\text{Rng}(f) = \{f(x) \mid x \in \text{Dom}(x)\}.$$  

Definition (pg. 118). Let $f$ and $g$ be functions. The **composition** of $g$ with $f$ is the function $g \circ f$ with domain

$$\text{Dom}(g \circ f) = \{x \mid x \in \text{Dom}(f) \text{ and } f(x) \in \text{Dom}(g)\}$$

and for all $x \in \text{Dom}(g \circ f)$ we set $(g \circ f)(x) = g(f(x))$.

Definition (pg. 119). Let $f : A \to B$ and let $C \subset A$. The **restriction** of $f$ to $C$ is the function $f|_C : C \to B$ and for all $x \in C$ we have $f|_C(x) = f(x)$.

Definition (pg. 119). The function $f$ is a **surjection** from the set $A$ to the set $B$ if $f : A \to B$ and for all $b \in B$ there exists $a \in A$ such that $f(a) = b$.

Definition (pg. 119). The function $f$ is an **injection** from the set $A$ into the set $B$ if $f : A \to B$ and for all $x, y \in A$ if $f(x) = f(y)$ then $x = y$.

Definition (pg. 126). If the function $f$ is an injection from the set $A$ into the set $B$ then the **inverse function** is

$$f^{-1} = \{(y, x) \mid (x, y) \in f\}.$$  

Definition (pg. 121). The function $f$ is a **bijection** from the set $A$ to the set $B$ if $f : A \to B$, $f$ is an injection from the set $A$ into the set $B$ and $f$ is a surjection from the set $A$ to the set $B$.

Definition (pg. 127). An **infinite sequence** in the set $A$ is a function $f : \mathbb{N} \to A$. If $f : \mathbb{N} \to A$ and for all $n \in \mathbb{N}$ we have $a_n = f(n)$ we usually denote the sequence given by $f$ as $(a_n)_{n \in \mathbb{N}}$.

Definition (pg. 128). If $A$ and $B$ are sets then the **set of all functions from $A$ to $B$** is denoted

$$B^A = \{f \mid f : A \to B\}.$$  

Definition (pg. 130). Let $A$ be a set. A **family of sets indexed by** $A$ is a function $f$ with $\text{Dom}(f) = A$ such that $f(\alpha)$ is a set for each $\alpha \in A$. If $f$ is a family of sets indexed by $A$ and for all $\alpha \in A$ we have $B_\alpha = f(\alpha)$ we usually denote the family of sets indexed by $A$ given by $f$ as $(B_\alpha)_{\alpha \in A}$.

Definition (pg. 130). Let $(B_\alpha)_{\alpha \in A}$ be a family of sets indexed by the set $A$. The **union** of $(B_\alpha)_{\alpha \in A}$ is the set

$$\bigcup_{\alpha \in A} B_\alpha = \{x \mid \text{There exists } \alpha \in A \text{ such that } x \in B_\alpha\}.$$
Definition (pg. 130). Let \((B_\alpha)_{\alpha \in A}\) be a family of sets indexed by the nonempty set \(A\). The **intersection** of \((B_\alpha)_{\alpha \in A}\) is the set \[\bigcap_{\alpha \in A} B_\alpha = \{x \mid \text{For all } \alpha \in A \text{ we have } x \in B_\alpha\} .\]

Definition (pg. 131). Let \(f\) be a function and \(A \subset \text{Dom}(f)\). The **image** of \(A\) under \(f\) is the set \[f[A] = \{f(x) \mid x \in A\} .\]

Definition (pg. 133). Let \(f\) be a function and \(B\) be a set. The **preimage** of \(B\) under \(f\) is the set \[f^{-1}[B] = \{x \in \text{Dom}(f) \mid f(x) \in B\} .\]

Definition (pg. 138). Let \(A\) and \(B\) be sets. The set \(A\) is **equinumerous** with the set \(B\) if there is a bijection from \(A\) to \(B\).

Definition (pg. 138). Let \(A\) be a set and \(n \in \omega\). The set \(A\) has **\(n\) elements** if there is a bijection from the set \(\{1, 2, 3, \cdots, n\}\) to \(A\).

Definition (pg. 139). The set \(A\) is **finite** if there exists \(n \in \omega\) such that \(A\) has \(n\) elements.

Definition (pg. 139). The set \(A\) is **infinite** if \(A\) is not finite.

Definition (pg. 144). If \(A\) is a set and \(k \in \omega\) then the set of all subsets of \(A\) with \(k\) elements is denoted \[\mathcal{P}_k(A) = \{S \subset A \mid S \text{ has } k \text{ elements }\}\].

Definition (pg. 157). The set \(A\) is **denumerable** (or **countably infinite**) if \(A\) is equinumerous with \(\mathbb{N}\).**

Definition (pg. 157). The set \(A\) is **countable** if \(A\) is finite or denumerable.

Definition (pg. 157). The set \(A\) is **uncountable** if \(A\) is not countable.