Math 3345 – April 18, 2014

Important definitions

Definition (pg. 38)**.** n is an **even number** if there is $k \in \mathbf{Z}$ such that $n = 2k$.

Definition (pg. 38)**.** n is an **odd number** if there is $k \in \mathbf{Z}$ such that $n = 2k + 1$.

Definition (pg. 41)**.** q is a **rational number** if there is $a \in \mathbf{Z}$ and $b \in \mathbf{Z}$ such that $b \neq 0$ and $q = \frac{a}{b}$.

Definition (pg. 42)**.** q is an **irrational number** if $x \in \mathbf{R}$ and x is not rational.

Definition (pg. 43)**.** Let $d, x \in \mathbf{Z}$. d **divides** x (or x is **divisible by** d) if there is $k \in \mathbf{Z}$ such that $x = dk$.

Definition (pg. 45)**.** Let $x \in \mathbf{N}$. x is a **prime number** if $x \neq 1$ and for all $a, b \in \mathbf{N}$ if $x = ab$ then $a = 1$ or $b = 1$.

Definition (pg. 48)**.** Let $a, b, m \in \mathbf{Z}$. a is **congruent to** b **modulo** m ($a \equiv b \mod m$) if m divides $b - a$.

Definition (pg. 58)**.** For all $n, k \in \boldsymbol{\omega}$ then the **binomial coefficients** $\binom{n}{k}$ satisfy the following two rules:

1. For all $n \in \boldsymbol{\omega}$ we have $\binom{n}{0} = \binom{n}{n} = 1$.
2. For all $n, k \in \mathbf{N}$ if $k \leq n$ then $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Definition (pg. 99)**.** The **empty set** $\varnothing$ is the set such that for all x we have $x \notin \varnothing$.

Definition (Lecture 22)**.** Let A and B be sets. A **equals** B ($A = B$) if for all x we have $x \in A$ if and only if $x \in B$.

Definition (pg. 100)**.** Let A and B be sets. A is a **subset** of B ($A \subset B$) if for all x if $x \in A$ then $x \in B$.

Definition (pg. 100)**.** Let A and B be sets. A is a **proper subset** of B ($A \subsetneq B$) if $A \subset B$ and $A \neq B$.

Definition (pg. 101)**.** Let A and B be sets. The **union** of the sets A and B is the set

$$A \cup B = \{x \,|\, x \in A \text{ or } x \in B\}.$$

Definition (pg. 101)**.** Let A and B be sets. The **intersection** of the sets A and B is the set

$$A \cap B = \{x \,|\, x \in A \text{ and } x \in B\}.$$

Definition (pg. 101)**.** Let A and S be sets. The **relative complement** of the set A in the set S is the set

$$S \setminus A = \{x \,|\, x \in S \text{ and } x \notin A\}.$$

Definition (pg. 105)**.** Let A and B be sets. The sets A and B are **disjoint** if $A \cap B = \varnothing$.

Definition (pg. 108)**.** Let $\mathcal{A}$ be a set of sets. The **union** of $\mathcal{A}$ is the set

$$\bigcup \mathcal{A} = \{x \,|\, \text{There exists } A \in \mathcal{A} \text{ such that } x \in A\}.$$

Definition (pg. 108)**.** Let $\mathcal{A}$ be a nonempty set of sets. The **intersection** of $\mathcal{A}$ is the set

$$\bigcap \mathcal{A} = \{x \,|\, \text{For all } A \in \mathcal{A} \text{ such that } x \in A\}.$$

Definition (pg. 109)**.** Let A be a set. The **power set** of A is the set

$$\mathcal{P}(A) = \{S \,|\, S \subset A\}.$$

Definition (pg. 110)**.** **Ordered pairs** (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.

Definition (pg. 112)**.** Let A and B be sets. The **Cartesian product** of A and B is the set

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$

Definition (pg. 126)**.** Let A and B be sets. A **function** f from A to B is a subset $f \subset A \times B$ satisfying that for all $a \in A$ there is a unique $b \in B$ such that $(a, b) \in f$ (we usually write $f(a) = b$ instead of $(a, b) \in f$).

Definition (pg. 113)**.** Let A and B be sets. If $f : A \to B$ then the **domain** of f is the set $\mathrm{Dom}(f) = A$.

Definition (pg. 114)**.** Let A be a set. The **identity function** on A is the function $\mathrm{id}_A : A \to A$ satisfying that for all $a \in A$ we have $\mathrm{id}_A(a) = a$.

Definition (pg. 114)**.** Let $A \subset B$. The **inclusion function** from A into B is the function $i : A \to B$ satisfying that for all $a \in A$ we have $i(a) = a$.

Definition (pg. 114)**.** Let A be a set. The **empty function** $\varnothing$ into A is the unique function satisfying $\varnothing : \varnothing \to A$.

Definition (pg. 115)**.** Let A and B be a sets. The **projection functions** are the functions π_A and π_B satisfying $\pi_A : A \times B \to A$ and $\pi_B : A \times B \to B$ and for all $a \in A$ and $b \in B$ we have $\pi_A(a, b) = a$ and $\pi_B(a, b) = b$.

Definition (pg. 117)**.** Let f be a function. The **range** of f is the set

$$\mathrm{Rng}(f) = \{f(x) \mid x \in \mathrm{Dom}(x)\}$$

Definition (pg. 118)**.** Let f and g be functions. The **composition** of g with f is the function $g \circ f$ with domain

$$\mathrm{Dom}(g \circ f) = \{x \mid x \in \mathrm{Dom}(f) \text{ and } f(x) \in \mathrm{Dom}(g)\}$$

and for all $x \in \mathrm{Dom}(g \circ f)$ we set $(g \circ f)(x) = g(f(x))$.

Definition (pg. 119)**.** Let $f : A \to B$ and let $C \subset A$. The **restriction** of f to C is the function $f|_C$ satisfying $f|_C : C \to B$ and for all $x \in C$ we have $f|_C(x) = f(x)$.

Definition (pg. 119)**.** The function f is a **surjection** from the set A to the set B if $f : A \to B$ and for all $b \in B$ there exists $a \in A$ such that $f(a) = b$.

Definition (pg. 119)**.** The function f is an **injection** from the set A into the set B if $f : A \to B$ and for all $x, y \in A$ if $f(x) = f(y)$ then $x = y$.

Definition (pg. 126)**.** If the function f is an injection from the set A into the set B then the **inverse function** is

$$f^{-1} = \{(y, x) \mid (x, y) \in f\}$$

Definition (pg. 121)**.** The function f is a **bijection** from the set A to the set B if $f : A \to B$, f is an injection from the set A into the set B and f is a surjection from the set A to the set B.

Definition (pg. 127)**.** An **infinite sequence** in the set A is a function $f : \mathbf{N} \to A$. If $f : \mathbf{N} \to A$ and for all $n \in \mathbf{N}$ we have $a_n = f(n)$ we usually denote the sequence given by f as $(a_n)_{n \in \mathbf{N}}$.

Definition (pg. 128)**.** If A and B are sets then the **set of all functions from** A **to** B is denoted

$$B^A = \{f \mid f : A \to B\}.$$

Definition (pg. 130)**.** Let A be a set. A **family of sets indexed by** A is a function f with $\mathrm{Dom}(f) = A$ such that $f(\alpha)$ is a set for each $\alpha \in A$. If f is a family of sets indexed by A and for all $\alpha \in A$ we have $B_\alpha = f(\alpha)$ we usually denote the family of sets indexed by A given by f as $(B_\alpha)_{\alpha \in A}$.

Definition (pg. 130)**.** Let $(B_\alpha)_{\alpha \in A}$ be a family of sets indexed by the set A. The **union** of $(B_\alpha)_{\alpha \in A}$ is the set

$$\bigcup_{\alpha \in A} B_\alpha = \{x \mid \text{There exists } \alpha \in A \text{ such that } x \in B_\alpha\}.$$

Definition (pg. 130)**.** Let $(B_\alpha)_{\alpha\in A}$ be a family of sets indexed by the nonempty set A. The **intersection** of $(B_\alpha)_{\alpha\in A}$ is the set

$$\bigcap_{\alpha\in A} B_\alpha = \{x \,|\, \text{For all } \alpha \in A \text{ we have } x \in B_\alpha\}.$$

Definition (pg. 131)**.** Let f be a function and $A \subset \mathrm{Dom}(f)$. The **image** of A under f is the set

$$f[A] = \{f(x) \,|\, x \in A\}.$$

Definition (pg. 133)**.** Let f be a function and B be a set. The **preimage** of B under f is the set

$$f^{-1}[B] = \{x \in \mathrm{Dom}(f) \,|\, f(x) \in B\}.$$

Definition (pg. 138)**.** Let A and B be sets. The set A is **equinumerous** with the set B if there is a bijection from A to B.

Definition (pg. 138)**.** Let A be a set and $n \in \boldsymbol{\omega}$. The set A **has** n **elements** if there is a bijection from the set $\{1, 2, 3, \cdots, n\}$ to A.

Definition (pg. 139)**.** The set A is **finite** if there exists $n \in \boldsymbol{\omega}$ such that A has n elements.

Definition (pg. 139)**.** The set A is **infinite** if A is not finite.

Definition (pg. 144)**.** If A is a set and $k \in \boldsymbol{\omega}$ then the **set of all subsets of** A **with** k **elements** is denoted

$$\mathcal{P}_k(A) = \{S \subset A \,|\, S \text{ has } k \text{ elements }\}.$$

Definition (pg. 157)**.** The set A is **denumerable** (or **countably infinite**) if A is equinumerous with $\mathbf{N}$.

Definition (pg. 157)**.** The set A is **countable** if A is finite or denumerable.

Definition (pg. 157)**.** The set A is **uncountable** if A is not countable.