# Math 3345
# Fundamentals of Higher Mathematics

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## Course Info

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text Falkner, *The Fundamentals of Higher Mathematics*

### Reading for Friday, January 10

pgs. 1-5

### HW1 Due Monday, January 13

- Section 2 Exercises: 1a, 2b, 3

## Epistemology

### Definition 1 (Epistemology)

**Epistemology** is the study of the nature of knowledge and how it is aquired.

### Question 2

*What are some effective methods for expanding human knowledge?*

- Scientific method
- Exploration & Observation
- Art & Fiction
- **Logical inference**





## What is mathematics?

- Model our universe and others through **axioms** and **proofs** and **theorems**.
- Axioms are assumptions

  For example:

  Axiom 1 All mammals are animals

  Axiom 2 All dogs are mammals
- Proofs use **propositional calculus** to combine axioms and previously proven theorems to establish new theorems.

  For example:

  Theorem All dogs are animals.

  Proof: By Axiom 2 all dogs are mammals. By Axiom 1 all mammals are animals. Hence all dogs are mammals. □

# What is mathematics?

- Mathematicians don't set around all day adding big numbers! (That's called arithmetic and we're generally bad at it)
- We sit around all day trying to prove **new theorems**. (or maybe old theorems in new ways)
- It usually takes $\sim$10 years of study to even understand the theorems mathematicians are proving today.
- Until then you learn about theorems of other mathematicans.
- Mathematics is useful for proving things about
  - numbers (and sets of numbers)
  - functions (and sets of functions)
  - other mathematical objects (groups, rings, fields, vector spaces, topological spaces, proofs, ...)
- Here are some statements of theorems that you can understand now:





### Theorem 3 (Pythagoreans 400-5 B.C.)

*There is no rational number $x$ such that $x^2 = 2$.*

Rummored that Hippasus was thrown overboard for divulging proof.

### Theorem 4 (Cantor 1874)

*There are just as many even integers as integers and rational numbers.*

### Theorem 5 (Cantor 1874)

*There more real numbers than integers.*

### Theorem 6 (Gödel 1931)

*There are true statements about the integers that cannot be proven.*

### Theorem 7 (Gödel 1931)

*The axioms of modern math cannot be proven consistent.*

## Theorem 8 (Wiles 1995)

*For $n > 2$ there are no positive integers $a, b, c$ such that $a^n + b^n = c^n$.*

*(Actually Wiles proved the Taniyama-Shimura-Weil Conjecture which is one of those incomprehensible math statements)*

## Theorem 9 (Perlman 2003)

*The 3-dimensional analog of the sphere is the only "simple" 3-dimensional "space."*

*(Again Perlman proved much more but it's hard to explain without a year of graduate math.)*

*(Perlman is an odd guy. He turned down the million dollar Millenium Prize and the $15,000 Fields Medal.)*





- Euclid derived vibrant ecosystem of theorems concerning planar geometry from 5 axioms in 300 B.C.
- Peano characterized the natural numbers

$$\mathbf{N} = \{1, 2, 3, 4, \cdots\}$$

  with 9 axioms in 1889.
- Modern math which subsumes above is based on ZFC (the Zermelo-Fraenkel axioms of set theory plus the axiom of choice)
- The 10 axioms of ZFC are mostly simple observations about sets.
- For example the first axiom of ZFC is the **Axiom of Extensionality:**

  If sets $A$ and $B$ have the same elements then $A = B$.

- Not useful to give the axioms yet.
- We'll return to set theory in Section 10.

- ZFC = Axioms of Set Theory
- Within ZFC we can prove all of Euclid, Peano + almost all modern math
- If you want more axioms then go into logic or philosophy (That's not a dismissal of either!)
- UPSHOT: Solid foundation in Set Theory needed to do math.