# Math 3345
# Fundamentals of Higher Mathematics

Nathan Broaddus

Ohio State University

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## Course Info

### Instructor - Nathan Broaddus

webpage https://people.math.osu.edu/broaddus.9/3345

office hours Mondays and Wednesdays 10:1am0-11am & 1pm-1:45pm?

### Reading for Monday, January 13

pgs. 6-9

### HW2 Due Wednesday, January 15

- Section 2 Exercises: 5a, 5b, 6

- Basic unit of arithmetic is number
- Basic unit of logic is **sentence (or statement)**
- A sentence must have a subject & verb.

### Example 10 (Statements)

1. The sun rises in the west.
2. $20 < 0$.
3. $y = 3x + 4$.

- Sometimes **truth value** (T or F) of sentence is clear. Sometimes depends on context.

### Example 11 (Not statements)

1. $3x$
2. $30 - 5$.





## Atomic Statements

### Example 12 (Truth value)

| Statement | Truth value |
|---|---|
| 3 is even | F |
| $1 + 2 = 3$ | T |
| $1 > 2$ | F |
| $x > 9$ | undetermined (see bound vs. unbound variables §3) |

- We represent an unknown or unspecified number with a variable (e.g. $x$)
- We will represent an unknown or unspecified statement with a variable (usually $P$, $Q$, or $R$)
- Sometimes a statement will depend on another variable
  - For example:
    $$P(x) = \text{“}x \text{ is an even integer.”}$$
  - Then
    $$P(3) = \text{“3 is an even integer.”}$$
    Has truth value F and
    $$P(-20) = \text{“}-20 \text{ is an even integer.”}$$
    Has truth value T.
- We can add, subtract, multiply or divide numbers.
- What operations can we do on statements?





## Operations on statements

- negation $\neg P$
- conjunction (“and”) $P \wedge Q$
- disjunction (“or”) $P \vee Q$
- implication (“If ... then” or “implies”) $P \Rightarrow Q$
- biconditional (“if and only if”) $P \Leftrightarrow Q$
- We give meaning of above with **truth tables**.

# Negation

Truth table for $\neg P$ (read "not $P$")

| $P$ | $\neg P$ |
|---|---|
| T | F |
| F | T |

## Example 13 (Negation)

1. If $P =$ "3 is even" then $\neg P =$"It is not true that 3 is even". Note that $P$ is false so $\neg P$ must be true.
2. If $Q =$ "Irene knows everyone" then $\neg Q =$"It is not true that Irene knows everyone".





# Conjunction

Truth table for $P \wedge Q$ (read "$P$ and $Q$")

| $P$ | $Q$ | $P \wedge Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

## Example 14 (Conjunction)

1. If $P =$ "3 is even" and $Q =$ "4 is even" then
$$P \wedge Q = \text{“3 is even and 4 is even”}$$
which is false.
2. If $P =$ "6 is even" and $Q =$ "4 is even" then
$$P \wedge Q = \text{“6 is even and 4 is even”}$$
which is true.

# Disjunction

Truth table for $P \vee Q$ (read "$P$ or $Q$")

| $P$ | $Q$ | $P \vee Q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## Example 15 (Disjunction)

1. If $P =$ "3 is odd" or $Q =$ "9 is odd" then
$$P \vee Q = \text{"3 is odd or 9 is odd"}$$
which is **true**.
2. If $P =$ "3 is odd" or $Q =$ "9 is even" then
$$P \vee Q = \text{"3 is odd or 9 is even"}$$
which is **true**.



Lecture 2 - 1/10/2014 Propositional Calculus Computing truth tables Logical equivalence

# Implication

Truth table for $P \Rightarrow Q$ (read "If $P$ then $Q$" or "$P$ implies $Q$")

| $P$ | $Q$ | $P \Rightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

## Example 16 (Implication)

1. If $P =$ "3 is odd" or $Q =$ "9 is odd" then
$$P \Rightarrow Q = \text{"3 is odd implies 9 is odd"}$$
which is **true**.
2. If $P =$ "3 is odd" or $Q =$ "9 is even" then
$$P \Rightarrow Q = \text{"If 3 is odd then 9 is even"}$$
which is **false**.

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# Biconditionals

Truth table for $P \Leftrightarrow Q$ (read "$P$ if and only if $Q$")

| $P$ | $Q$ | $P \Leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

## Example 17 (Biconditionals)

1. If $P =$ "3 is odd" or $Q =$ "9 is odd" then
$$P \Leftrightarrow Q = \text{"3 is odd if and only if 9 is odd"}$$
which is **true**.
2. If $P =$ "3 is odd" or $Q =$ "9 is even" then
$$P \Leftrightarrow Q = \text{"If 3 is odd if and only if 9 is even"}$$
which is **false**.





# Computing truth tables

To compute a truth table for a statement (e.g. $(\neg P) \vee Q$) involving unknown statement $P$, $Q$

1. Left 2 columns should list all 4 possibilities for truth or falseness of $P$ and $Q$.

| $P$ | $Q$ | |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

2. Now list substatements which are needed to evaluate full statement across top. In our case we need $\neg P$ and then $(\neg P) \vee Q$.

| $P$ | $Q$ | $\neg P$ | $(\neg P) \vee Q$ |
|---|---|---|---|
| T | T | | |
| T | F | | |
| F | T | | |
| F | F | | |

# Computing truth tables

Now fill in empty columns in logical order

3.

| $P$ | $Q$ | $\neg P$ | $(\neg P) \vee Q$ |
|---|---|---|---|
| T | T | F | |
| T | F | F | |
| F | T | T | |
| F | F | T | |

4.

| $P$ | $Q$ | $\neg P$ | $(\neg P) \vee Q$ |
|---|---|---|---|
| T | T | F | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |





# Computing truth tables

## Example 18 (Computing truth tables)

1. What should the column headings for a truth table for $P \Rightarrow (P \wedge Q)$ be?

$$P, Q, P \wedge Q, (\neg P) \vee Q$$

2. Fill in the truth table for $P \Rightarrow (P \wedge Q)$.

| $P$ | $Q$ | $P \wedge Q$ | $P \Rightarrow (P \wedge Q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | F | T |
| F | F | F | T |

# Logical equivalence

- Compute truth table for statement $\neg\neg P$:

| $P$ | $\neg P$ | $\neg\neg P$ |
|---|---|---|
| T | F | T |
| F | T | F |

- Notice that columns headed $P$ and $\neg\neg P$ has **same** entries.
- We say that the statments $P$ and $\neg\neg P$ are **logically equivalent** and write

$$P \equiv \neg\neg P$$





# Logical equivalence

## Example 19 (Logical equivalence)

Show that $P \Rightarrow Q$ is logically equivalent to $P \Rightarrow (P \wedge Q)$ using a truth table.

1. Compute truth table for $P \Rightarrow Q$ and $P \Rightarrow (P \wedge Q)$ (we'll do both in one):

| $P$ | $Q$ | $P \Rightarrow Q$ | $P \wedge Q$ | $P \Rightarrow (P \wedge Q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | F | T | F | T |

2. Notice that column 3 and column 5 are identical.
3. Hence $P \Rightarrow Q \equiv P \Rightarrow (P \wedge Q)$

# Logical equivalence

## Theorem 20 (DeMorgan's Laws)

1. $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$
2. $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

## Proof of 1.

| $P$ | $Q$ | $P \wedge Q$ | $\neg(P \wedge Q)$ | $\neg P$ | $\neg Q$ | $(\neg P) \vee (\neg Q)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | F | T | F | T | T |
| F | T | F | T | T | F | T |
| F | F | F | T | T | T | T |

Note that columns 4 and 7 match. Hence $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$. □