Math 3345
Fundamentals of Higher Mathematics

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Course Info

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Reading for Friday, January 17

pgs. 15-18

HW4 Due Wednesday, January 22

- Section 2 Exercises: 7a, 13, 14

Warm-up Problems

Problem 27

Solution to Excercise 5b

Problem 28

For what real numbers x is the statement

$$P(x) = 1 \leq x < 2$$

false?

Note

$$1 \leq x < 2 \text{ means } (1 \leq x) \wedge (x < 2).$$

Warm-up Problems

Solution to Problem 28

$$\begin{aligned} \neg(1 \leq x < 2) &\equiv \neg\big((1 \leq x) \wedge (x < 2)\big) \\ &\equiv \neg(1 \leq x) \vee \neg(x < 2) \\ &\equiv (1 > x) \vee (x \geq 2) \end{aligned}$$

Thus $P(x)$ is false if the real number x is less than 1 or greater than or equal to 2.

More on implications

Definition 29 (Converse and Contrapositive)

Given an implication $P \Rightarrow Q$ the **converse** is the statement $Q \Rightarrow P$ and the **contrapositive** is the statement $\neg Q \Rightarrow \neg P$.

Truth table for $P \Rightarrow Q$, $Q \Rightarrow P$ and $\neg Q \Rightarrow \neg P$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$
T	T	T	T	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Theorem 30

$(P \Rightarrow Q) \equiv (\neg Q \Rightarrow \neg P)$

Warning

A statement is not logically equivalent to its converse.

More on implications

Example 31

- Let Q be the statement "If today is Monday then tomorrow is Tuesday."
- Then converse of Q is "If tomorrow is Tuesday then today is Monday."
- The contrapositive of Q is "If tomorrow is not Tuesday then today is not Monday."
- In this case all statements are true.

More on implications

Example 32

- Let $P(x)$ be the statement "If x is an integer then x is a real number."
- Then converse of $P(x)$ is $Q(x) =$"If x is a real number then x is an integer."
- The contrapositive of $P(x)$ is $R(x) =$"If x is not a real number then x is not an integer."
- In this case $P(x)$ and $R(x)$ are true no matter the value of x.
- But the converse $Q(x)$ is only true if x is an integer or x is not a real number.

Example 33

- Let R be the statement "$(1 + 2 = 5) \wedge (1 < 2)$".
- Then R has no converse or contrapositive since it is not an implication.

More on tautologies

Example 34 (Tautologies)

The following statements are all tautologies:

1. $P \vee \neg P$ Law of the excluded middle
2. $P \Rightarrow P$
3. $(P \wedge Q) \Rightarrow P$ Example 2.21 in book
4. $Q \Rightarrow (P \vee Q)$ Exercise 13 (HW4 problem)
5. $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ Modus ponens
6. $Q \Rightarrow (P \Rightarrow Q)$

All the above may be proven with truth tables.

More on tautologies

Definition 35 (Contradiction)

A **contradiction** is a sentence of the form $Q \wedge \neg Q$ and hence always false.

Warning

Contradictions are always of this form.

Example 36 (Contradictions)

The following statements are all contradictions

1. $1 = 2 \wedge 1 \neq 2$
2. $(x \text{ is even}) \wedge (x \text{ is not even})$

All the above may be proven with truth tables.

Conditional proof

Method of conditional proof

In order to prove the conditional statement $A \Rightarrow B$ assume that A is true and under that assumption show that B is true.

Theorem 37

$Q \Rightarrow (P \Rightarrow Q)$ is a tautology.

Proof.

We will use the method of conditional proof.

[A1] Assume that Q is true.

Then for any statement P the statement $P \Rightarrow Q$ holds.

Discharging [A1] we have proven that $Q \Rightarrow (P \Rightarrow Q)$ is true without any assumptions. Thus $Q \Rightarrow (P \Rightarrow Q)$ is a tautology. □

Conditional proof

A proof using the method of conditional proof should always look like:

Theorem 38

$A \Rightarrow B$

Proof.

We will use the method of conditional proof.

A1 Assume A is true.

Then

⋮

thus B is true.

Discharging A1 we have proven that $A \Rightarrow B$ is true without any assumptions. □

Warning

Method of conditional proof can only be used to prove conditional statements.