Math 3345
Fundamentals of Higher Mathematics

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January 22, 2014

Course Info

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Reading for Friday, January 24

pgs. 24-27

Quiz 1 Friday, January 24

All of Section 2

Theorem 41

$[(P \wedge Q) \Rightarrow R] \Rightarrow [P \Rightarrow (Q \Rightarrow R)]$ *is a tautology.*

Proof.

We will use the method of conditional proof.

A1 Suppose that $(P \wedge Q) \Rightarrow R$ is true.

A2 Suppose that P is true

A3 Suppose that Q is true.

Then By assumptions A2 and A3 $P \wedge Q$ is true. It follows by A1 that R must be true.

Discharging A3 we have proven that $Q \Rightarrow R$ under assumptions A1 and A2.

Discharging A2 we have proven that $P \Rightarrow (Q \Rightarrow R)$ under assumption A1.

Discharging A1 we have proven that $[(P \wedge Q) \Rightarrow R] \Rightarrow [P \Rightarrow (Q \Rightarrow R)]$ is true without any assumptions. □

Proof by contradiction

Method of proof by contradiction

In order to prove statement A prove that $\neg A \Rightarrow (B \wedge \neg B)$ is true for some statement B.

Theorem 42

There is no largest natural number.

Proof.

We will use the method of proof by contradiction.

A1 Suppose there is a largest natural number.

Then let n be the largest natural number. It follows that $n + 1$ is a natural number and $n + 1 > n$. Hence n is not the largest natural number. Thus n is the largest natural number and n is not the largest natural number. This is a contradiction.

Discharging A1 we have proven that there is no largest natural number. □

Proof by contradiction

A proof using the method of proof by contradiction should always look like:

Theorem 43

A is true.

Proof.

We will use the method of proof by contradiction.

[A1] Assume $\neg A$ is true.
Then
⋮
thus $B \wedge \neg B$ is true which is a contradiction.

Discharging [A1] we have proven that A is true without assumptions. □

Proof by contraposition

Method of proof by contraposition

In order to prove the statement $A \Rightarrow B$ prove that $\neg B \Rightarrow \neg A$ is true.

Theorem 44

Let x be an integer. If x^2 is even then x is even.

Proof.

Assume that x is an integer. We will use the method of proof by contraposition.

[A1] Suppose x is not even.
Then x is odd. Thus x^2 is an odd integer. Therefore x^2 is not even.

Discharging [A1] we have proven that under the assumption that x is an integer, if x is even then x^2 is even. □

Proof by contraposition

A proof using the method of proof by contraposition should always look like:

Theorem 45

$A \Rightarrow B$ is true.

Proof.

We will use the method of proof by contraposition.

A1 Assume $\neg B$ is true.

Then

$\vdots$

thus $\neg A$ is true.

Discharging A1 we have proven that $A \Rightarrow B$ is true without assumptions. □

Methods of proof

For the following statements give a setup for a proof by (a) the method of conditional proof (b) the method of contraposition (c) by contradiction.

Theorem 46

If x is rational then $x^2 \neq 2$

(a) Proof: Conditional proof. A1 Assume that x is rational.

(b) Proof: Proof by contraposition. A1 Assume that $x^2 = 2$.

(c) Proof: Proof by contradiction. A1 Assume that it is not true that if x is rational then $x^2 \neq 2$.

Methods of proof

For the following statements give a setup for a proof by (a) the method of conditional proof (b) the method of contraposition (c) by contradiction.

Theorem 47

If n and k are odd integers then nk is an odd integer.

(a) Proof: Conditional proof. A1 Assume that n and k are odd integers.

(b) Proof: Proof by contraposition. A1 Assume that nk is not an odd integer.

(c) Proof: Proof by contradiction. A1 Assume that it is not true that if n and k are odd integers then nk is an odd integer.

Methods of proof

For the following statements give a setup for a proof by (a) the method of conditional proof (b) the method of contraposition (c) by contradiction.

Theorem 48

There are infinitely many prime numbers.

(a) Can't use conditional proof! (Statement is not an implication)

(b) Can't use proof by contraposition! (Statement is not an implication)

(c) Proof: Proof by contradiction. A1 Assume that it is not true that there are infinitely many prime numbers.