Math 3345
Fundamentals of Higher Mathematics

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Course Info

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HW6 Due Wednesday, January 29

- Section 2 Exercises: 20, 28b, 30

Bonus Problems (Hand in any time before April 18)

Section 2 Exercises 18, 29, 34

Truth functions

- A **truth function** is a rule which associates a truth value to the truth value of a set number of variables.
- Two ways to define a truth function:
  1. With a formula:
  $$f(P, Q) = P \vee (Q \Leftrightarrow P)$$
  2. With a truth table:

P	Q	$g(P, Q)$
T	T	T
T	F	T
F	T	F
F	F	T

- Notice above definitions give same truth functions so $f = g$.

Truth functions

- We saw in Lecture 3 why any truth function can be written as a formula using just $\wedge$, $\vee$ and $\neg$.
- For example: Give a sentence whose trinary truth function has the following truth table:

P	Q	R	$g(P, Q, R)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

- $g(P, Q, R) = (P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$
- Recall we can eliminate $\vee$ since $P \vee Q \equiv \neg\neg(P \vee Q) \equiv \neg(\neg P \wedge \neg Q)$.

Truth functions

- In fact the operations $\neg$, $\wedge$ are **functionally complete**
- Any n-ary truth function can be given as a formula using only the operations $\neg$ and $\wedge$.
- Actually there is a single binary function $|$ called **nand** which is functionally complete.
- Define
$$P|Q = \neg(P \wedge Q)$$
- Then
$$\neg P \equiv \neg(P \wedge P) \equiv P|P$$
-
$$P \wedge Q \equiv \neg\neg(P \wedge Q) \equiv \neg(P|Q) \equiv (P|Q)|(P|Q)$$
- The binary function $\downarrow$ called **nor** is also functionally complete. Can you express $\neg$ and $\wedge$ using only $\downarrow$?