# Math 3345
# Fundamentals of Higher Mathematics

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## Course Info

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### Reading for Wednesday, January 27

pgs. 28-33

### HW7 Due Friday, January 31

- Section 3 Exercises: 1bek, 2a, 4

### Bonus Problems (Hand in any time before April 18)

Section 2 Exercises 18, 29, 34

# Quantifiers

We now introduce two key symbols into our logical statements:

## Universal quantifier

$$(\forall x)P(x)$$

means

For all $x$, P(x) is true.

## Existential quantifier

$$(\exists x)P(x)$$

means

There exists $x$ such that P(x) is true.





# Universe of discourse

- It's hard to make sense of a sentence like

  $$(\forall x)x^2 \geq 0$$

  unless we have some restriction on what values $x$ can take.
- We must specify the **universe of discourse** for quantified sentences.
- This is a collection containing all values which we will consider for $x$.
- Here are some key collections which we will want to be in our universe of discourse:
  - The **natural numbers N** $= \{1, 2, 3, \cdots\}$
  - The **whole numbers** $\boldsymbol{\omega} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$
  - The **integers Z** $= \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$
  - The **rationals Q** (see Section 4 for definition)
  - The **reals R** (not defined in this class but think "infinite decimals")
  - The **complex numbers C** (think "pairs of real numbers with funky multiplication rule")
  - The collection of all sets that can be proved to exist using axioms of ZFC (This is the universe of discourse for modern mathematics and hence the default unless another universe of discourse is specified).

# Subcollections of the universe of discourse

## The membership symbol $\in$

If $A$ is a subcollection of the universe of discourse

$$x \in A$$

means $x$ is in $A$.

- $$(\forall x \in A)P(x)$$
  is shorthand for $(\forall x)[(x \in A) \Rightarrow P(x)]$.
- $$(\exists x \in A)P(x)$$
  is shorthand for $(\exists x)[(x \in A) \wedge P(x)]$.





## Example 49

Decide if the following statements are true or false:

1. $(\forall x \in \mathbf{R})(x+1 > x)$ T
2. $(\exists x \in \mathbf{R})(x+1 > x)$ T
3. $(\forall y \in \mathbf{N})(y-2 = 8)$ F
4. $(\exists y \in \mathbf{N})(y-2 = 8)$ T
5. $(\forall s \in \mathbf{R})[(s^2 > 9) \Rightarrow (s > 3)]$ F
6. $(\exists s \in \mathbf{R})[(s^2 > 9) \Rightarrow (s > 3)]$ T

# More notation

- $$(\forall x > y)P(x)$$
  is shorthand for $(\forall x)\{[(x \in \mathbf{R}) \wedge (x > y)] \Rightarrow P(x)\}$.
- $$(\exists x \in A)P(x)$$
  is shorthand for $(\exists x > y)[(x \in \mathbf{R}) \wedge (x > y) \wedge P(x)]$.
- Same for $<$, $\geq$, $\leq$

## Example 50

Decide if the following statements are true or false:

1. $(\forall s > 0)[(s^2 > 9) \Rightarrow (s > 3)]$ T
2. $(\exists s < 0)[(s^2 > 9) \Rightarrow (s > 3)]$ T





# Examples and counterexamples

- To show that the existential statement $(\exists x)P(x)$ is true find a value for $x$ for which $P(x)$ is true. Such a value is called an **example**
- To show that the universal statement $(\forall x)P(x)$ is false find a value for $x$ for which $P(x)$ is false. Such a value is called a **counterexample**

## Example 51

Find an appropriate counterexample or example for the following statements:

1. $(\forall s \in \mathbf{R})[(s^2 > 9) \Rightarrow (s > 3)]$ counterexample: $s = -4$
2. $(\exists s \in \mathbf{R})[(s^2 > 9) \Rightarrow (s > 3)]$ example: $s = 4$
3. $(\forall s > 0)[(s^2 > 9) \Rightarrow (s > 3)]$ This statement is true so there is no counterexample
4. $(\exists s < 0)[(s^2 > 9) \Rightarrow (s > 3)]$ example: $s = -1$
5. $(\exists y \in \mathbf{N})(y - 2 = 8)$ example: $y = 10$

# Examples and counterexamples

**Use an example**

to show that the existential statement $(\exists x)P(x)$ is true.

**Use a counterexample**

to show that the universal statement $(\forall x)P(x)$ is false.





# "and" vs "for all"

- Suppose our universe discourse is finite (For example $\{0, 1, 2\}$)
- Then the universal statement

$$(\forall x)P(x)$$

is logically equivalent to

$$P(0) \wedge P(1) \wedge P(2)$$

- and the existential statement

$$(\exists x)P(x)$$

is logically equivalent to

$$P(0) \vee P(1) \vee P(2)$$