Math 3345
Fundamentals of Higher Mathematics

Nathan Broaddus

Ohio State University

March 21, 2014

Course Info

HW20 Due Wednesday, March 26

- Section 10 Exercises 8, 10, 14

Quiz 3 Friday, March 28 in class

Quiz 3 will cover pgs. 97-107

Warm-up Problems

Problem 1

Solution to Section 4 Exercise 20:

If x is a rational number such that $c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0 = 0$ where $n \in \mathbf{N}$ and $c_0, c_1, \cdots, c_{n-1}, c_n \in \mathbf{Z}$, then there are $a, b \in \mathbf{Z}$ such that $x = \frac{a}{b}$, a divides c_0 and b divides c_n.

Set Operations

Definition 2 (Set operations)

Let A and B be sets.

1. The **union** of A and B is the set

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

2. The **intersection** of A and B is the set

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

3. The **relative complement** of B in A is the set

$$A \setminus B = \{x | x \in A \text{ and } x \notin B\}$$

Set Operations

Example 3 (Set operations)

1. $$\{1,2,3\} \cup \{2,3,4\} = \{1,2,3,4\}$$
2. $$\{1,2,3\} \cap \{2,3,4\} = \{2,3\}$$
3. $$\{1,2,3\} \setminus \{2,3,4\} = \{1\}$$

Lecture 21 - 3/21/2014

Set Operations

Theorem 4

If A, B and C are sets such that $A \subset C$ an $B \subset C$ then $A \cup B \subset C$.

Proof.

Suppose A, B and C are sets such that $A \subset C$ an $B \subset C$. Suppose $x \in A \cup B$. Then $x \in A$ or $x \in B$.

Case 1 Suppose $x \in A$. By assumption $A \subset C$ so $x \in A \Rightarrow x \in C$. Hence $x \in C$.

Case 2 Suppose $x \in B$. By assumption $B \subset C$ so $x \in B \Rightarrow x \in C$. Hence $x \in C$.

We have shown that $(\forall x) x \in (A \cup B) \Rightarrow x \in C$. Hence $A \cup B \subset C$. □

Nathan Broaddus Fundamentals of Higher Math

Set Operations

Theorem 5 (De Morgan's Laws for sets)

Let A, B and S be sets. Then

1. $S \setminus (A \cup B) = (S \setminus A) \cap (S \setminus B)$.
2. $S \setminus (A \cap B) = (S \setminus A) \cup (S \setminus B)$.

Proof.

Proof of 2 in HW20.
Proof of 1: Let A, B and S be sets. Given x
$x \in S \setminus (A \cup B)$
iff $x \in S$ and $x \notin (A \cup B)$
iff $x \in S$ and $\neg(x \in A \cup B)$
iff $x \in S$ and $\neg(x \in A$ or $x \in B)$
iff $x \in S$ and $(x \notin A$ and $x \notin B)$
iff $(x \in S$ and $x \notin A)$ and $(x \in S$ and $x \notin B)$
iff $(x \in S \setminus A)$ and $(x \in S \setminus B)$
iff $x \in (S \setminus A) \cap (S \setminus B)$ □