Math 3345
Fundamentals of Higher Mathematics

Nathan Broaddus

Ohio State University

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Course Info

HW22 Due Wednesday, April 2

- Section 11 Exercises 1abc, 2, 4abc

Functions

Definition 1 (Function)

Let A and B be sets. We say that f is a **function** from the set A to the set B if $f \subset A \times B$ and for all $a \in A$ there is a unique $b \in B$ such that $(a, b) \in f$.

Notation

$$f : A \to B$$

means "f is a function from the set A to the set B".

Notation

$f(a) = b$ means $(a, b) \in f$.

Lecture 26 - 3/28/2014

Functions

Example 2 (Functions)

Let $A = \{1, 2\}$ and $B = \mathbf{N}$

1. Let
$$f = \{(1, 5), (2, 7)\}$$
Then f is a function from A to B and $f(1) = 5$ and $f(2) = 7$.
2. Let
$$g = \{(1, 6)\}$$
Then g is **not** function from A to B but g **is a function** from $\{1\}$ to B and $g(1) = 6$.
3. Let
$$h = \{(1, 6), (2, 7), (1, 3)\}$$
Then h is **not** function since there is not a **unique** $b \in \mathbf{N}$ such that $(1, b) \in h$.

Functions

Definition 3 (Domain and Range)

Given $f : A \to B$. The **domain** of f is the set

$$\text{dom}(f) = A$$

The **range** of f is the set

$$f(A) = \{f(a) | \, a \in A\}$$

Specifying functions

A function may be specified with a formula

Example 4 (Functions from formulas)

1. Let $f : \mathbf{R} \to \mathbf{R}$ be the function
$$f(x) = x^2 + 3x$$
means $f = \{(x, x^2 + 3x) \,|\, x \in \mathbf{R}\}$.
2. Let $g : \mathbf{R} \to \mathbf{R}$ be the function
$$g(x) = \begin{cases} 3, & x \leq 0 \\ x^3, & x > 0 \end{cases}$$
means $g = \{(y, 3) \,|\, y \leq 0\} \cup \{(y, y^3) \,|\, y > 0\}$.

Specifying functions

But **most functions have no formula**

Example 5 (Functions with no formula)

Let $A = \{x | x \text{ is a student at OSU}\}$. Let $h : A \to \omega$ be the function

$$h(x) = \text{ number of cars owned by } x$$

Then h is a function but it cannot be expressed with a formula.

Important functions

Functions on **R** specified by a formula come with an implicit domain.

Definition 6

1. Let A be a set. The **identity function** on A is the function $\text{id}_A : A \to A$ where for all $a \in A$

$$\text{id}_A(a) = a$$

2. Let A and B be a sets such that $A \subset B$. The **inclusion function** on A is the function $i : A \to B$ where for all $a \in A$

$$i(a) = a$$

Important functions

Functions on $\mathbf{R}$ specified by a formula come with an implicit domain.

Definition 7 (Projection functions)

Let A and B be a sets. The **coordinate projection functions** on $A \times B$ are the functions $\pi_A : A \times B \to A$ and $\pi_B : A \times B \to B$ where for all $a \in A$ and $b \in B$

$$\pi_A((a, b)) = a$$

and

$$\pi_B((a, b)) = b$$

Notation

If f is a function with domain $A \times B$ then we write $f(a, b)$ instead of $f((a, b))$ to avoid excessive parentheses. So for π_A and π_B above we would write $\pi_A(a, b) = a$ and $\pi_B(a, b) = b$.