Math 3345
Fundamentals of Higher Mathematics

Nathan Broaddus

Ohio State University

April 2, 2014

Course Info

HW24 Due Monday, April 7, 2014

- Section 11 Exercises 13a-c, 15, 19

Warm-up Problems

Problem 1

Solutions to Quiz 3

Problem 2

Solution to Section 11 Exercise 2:

Let $I : C[a, b] \to \mathbf{R}$ *be the function given by the formula*

$$I(f) = \int_a^b f(x)\,\mathrm{d}x$$

if $g(x) = c$ *for all* $x \in [a, b]$ *what is* $I(g)$*?*

Warm-up Problems

Problem 3

Suppose that $f : (0, \infty) \to \mathbf{R}$ *satifies*

$$\forall x, y \in (0, \infty)\ f(xy) = f(x) + f(y)$$

Then

1. $f(1) = 0$.
2. $\forall x \in (0, \infty)\, f(\frac{1}{x}) = -f(x)$
3. $\forall x, y \in (0, \infty)\, f(\frac{x}{y}) = f(x) - f(y)$

Surjections and Injections

Definition 4 (Surjection)

If A and B are sets and $f : A \to B$. Then f is a **surjection from A onto B** if for all $b \in B$ there is $a \in A$ such that $f(a) = b$.

Definition 5 (Injection)

A function f is an **injection** if for all $x, y \in \text{Dom}(f)$ if $f(x) = f(y)$ then $x = y$.

Example 6 (Surjections and Injections)

For each of the following functions decide if $f : A \to B$ is an injection, a surjection or both.

1. Let $A = \mathbf{R}$, $B = [0, \infty)$ and $f : A \to B$ be the function $f(x) = x^2$.
   - f is a surjection since if $y \in [0, \infty)$ then $f(\sqrt{y}) = (\sqrt{y})^2 = y$
   - f is not an injection since $f(-1) = (-1)^2 = 1 = 1^2 = f(1)$ but $-1 \neq 1$.
2. Let $A = (0, \infty)$, $B = \mathbf{R}$ and $f : A \to B$ be the function $f(x) = \ln x$.
   - f is a surjection since if $y \in \mathbf{R}$ then $e^y \in (0, \infty)$ and $f(e^y) = \ln(e^y) = y$.
   - f is an injection since if $x, y \in (0, \infty)$ and $f(x) = f(y)$ then $\ln x = \ln y$ so $e^{\ln x} = e^{\ln y}$ so $x = y$.
3. Let $A = \mathbf{R}$, $B = \mathbf{R}$ and $f : A \to B$ be the function $f(x) = x(x-1)(x-2)$.
   - f can be shown to be a surjection using the intermediate value theorem since f is continuous on $\mathbf{R}$ and $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = -\infty$.
   - f is not an injection since $f(0) = f(1)$ but $0 \neq 1$.

Surjections and Injections

Theorem 7

If $f : A \to B$ is a surjection and $g : B \to C$ is a surjection then $g \circ f : A \to C$ is a surjection.

Proof.

Suppose that $f : A \to B$ is a surjection and $g : B \to C$ is a surjection. Let $c \in C$. The function g is a surjection so there is $b \in B$ such that $g(b) = c$. The function f is a surjection so there is $a \in A$ such that $f(a) = b$. Thus

$$g \circ f(a) = g(f(a)) = g(b) = c$$

Therefore $g \circ f : A \to C$ is a surjection. □