Math 3345
Fundamentals of Higher Mathematics

Nathan Broaddus

Ohio State University

April 9, 2014

Warm-up Problems

Problem 1

Solution to Section 11 Exercise 22ab:

Let A, B, C and D be sets and suppose that $A \cap B = \varnothing$. Let g and h be functions such that $g : A \to C$ and $h : B \to D$. Let φ be the function satisfying $\varphi : A \cup B \to C \cup D$ such that for all $x \in A \cup B$

$$\varphi(x) = \begin{cases} g(x), & x \in A \\ h(x), & x \in B \end{cases}$$

1. *If g and h are surjections then φ is a surjection.*
2. *If C and D are disjoint and g and h are injections then φ is an injection.*

Comparing the size of sets

Definition 2 (Equinumerous)

Let A and B be sets. The set A and the set B are **equinumerous** if there is a bijection $f : A \to B$

Notation

$|A| = |B|$ means that the sets A and B are equinumerous.

Notation

"A and B have the same size" means that the sets A and B are equinumerous.

Comparing the size of sets

Definition 3 (Set of Size n)

Let $n \in \omega$. The set A **has** n **elements** if there is a bijection $f : \{1, 2, 3, \cdots, n\} \to A$.

Notation

$|A| = n$ means that the set A has n elements.

Notation

"A has size n" means that the sets A has n elements.

Comparing the size of sets

Definition 4 (Finite Set)

The set A is **finite** if there exists $n \in \omega$ such that $|A| = n$.

Definition 5 (Infinite Set)

The set A is **infinite** if it is not finite.

Comparing the size of sets

Example 6 (Equinumerous Sets)

1. The sets $\{1, 2\}$ and $\{\pi, \pi^2\}$ are equinumerous since the function $f : \{1, 2\} \to \{\pi, \pi^2\}$ with $f(1) = \pi$ and $f(2) = \pi^2$ gives a bijection.
2. The set $\mathbf{Z}$ is equinumerous with the set of even integers $B = \{2n \mid n \in \mathbf{Z}\}$ since the function $g : \mathbf{Z} \to B$ with $f(n) = 2n$ is a bijection. **Note:** B is a proper subset of $\mathbf{Z}$ but the sets B and $\mathbf{Z}$ have the same size!