

Practice Midterm 1

Instructions:

- You will have 55min for this midterm.
- Prove your answers correct unless instructed not to.
- Cross out all scratchwork from you final answer.
- You may use any theorems from class or the book unless instructed not to.

1. (20 points) Decide if the following statements are TRUE or FALSE and circle your answer. **You do NOT need to justify your answer.**

- | | | | |
|----------|----------|---|--|
| T | F | ? | (a) $(\forall x \in \mathbf{Q})(\exists y \in \mathbf{R})(\forall n \in \mathbf{N})x + y = 1.$ |
| T | F | ? | (b) $(\forall x \in \mathbf{Z})(\forall y \in \mathbf{Z})\{(x + y \text{ is odd}) \Rightarrow [(x \text{ is odd}) \vee (y \text{ is odd})]\}.$ |
| T | F | ? | (c) $(3 \text{ is even}) \Rightarrow (2 \text{ is even}).$ |
| T | F | ? | (d) $(\exists!x \in \mathbf{Z})(\forall y \in \mathbf{Z})xy = 0.$ |
| T | F | ? | (e) $(\forall y \in \mathbf{Z})(\exists!x \in \mathbf{Z})xy = 0.$ |

2. (10 points) Prove that the statement $(\exists x \in \mathbf{Z})(\exists y \in \mathbf{Z})(\exists n \in \mathbf{N})xy = x$ is true.

3. (10 points) Prove that the statement $(\exists!x \in \mathbf{Z})(\exists y \in \mathbf{Z})(\exists n \in \mathbf{N})xy = x$ is false.

4. (20 points) Prove that $[\neg(P \wedge Q)] \Rightarrow [(P \Rightarrow Q) \Rightarrow \neg P]$ is a tautology using **proof by contraposition**. Do not use multiple cases. State and discharge your assumptions explicitly.

5. (20 points) Prove that for all integers x if x is odd then $2x$ is even or 3 is even.

6. (20 points) Suppose that the statement P is both true and false. Using this supposition prove that $\sqrt{3}$ is irrational. **State your method of proof** explicitly.