Practice Midterm 1

Instructions:

• You will have 55 min for this midterm.
• Prove your answers correct unless instructed not to.
• Cross out all scratchwork from you final answer.
• You may use any theorems from class or the book unless instructed not to.

1. **(20 points)** Decide if the following statements are TRUE or FALSE and circle your answer. **You do NOT need to justify your answer.**

- (a) \( (\forall x \in \mathbb{Q})(\exists y \in \mathbb{R})(\forall n \in \mathbb{N})x + y = 1. \)
- (b) \( (\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})\{(x + y \text{ is odd}) \Rightarrow [(x \text{ is odd}) \lor (y \text{ is odd})]\}. \)
- (c) \( (3 \text{ is even}) \Rightarrow (2 \text{ is even}). \)
- (d) \( (\exists! x \in \mathbb{Z})(\forall y \in \mathbb{Z})xy = 0. \)
- (e) \( (\forall y \in \mathbb{Z})(\exists! x \in \mathbb{Z})xy = 0. \)

2. **(10 points)** Prove that the statement \( (\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\exists n \in \mathbb{N})xy = x \) is true.

3. **(10 points)** Prove that the statement \( (\exists! x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\exists n \in \mathbb{N})xy = x \) is false.

4. **(20 points)** Prove that \( \neg(P \land Q) \Rightarrow [(P \Rightarrow Q) \Rightarrow \neg P] \) is a tautology using **proof by contraposition.**

Do not use multiple cases. State and discharge your assumptions explicitly.

5. **(20 points)** Prove that for all integers \( x \) if \( x \) is odd then \( 2x \) is even or \( 3 \) is even.

6. **(20 points)** Suppose that the statement \( P \) is both true and false. Using this supposition prove that \( \sqrt{3} \) is irrational. **State your method of proof** explicitly.