Math 4552 – February 25, 2015 Due Wednesday, March 4, 2015

Homework 6

Reading: Chapter 4 Sections 41-47

Homework Problems:

1. Compute:

   (a) $\displaystyle\int_1^2 \cos(3\pi it) + it^2 - 5t \, \mathrm{d}t.$

   (b) $\displaystyle\int_1^2 \tfrac{\mathrm{d}}{\mathrm{d}t}\Big(\cos(3\pi it - 1) + it^2 - 5t\Big) \, \mathrm{d}t.$

2. Show that for all $m, n \in \mathbf{Z}$
$$\int_0^{2\pi} e^{im\theta} e^{in\theta} \, \mathrm{d}\theta = \begin{cases} 0, & m \neq n \\ 2\pi, & m = n \end{cases}$$

3. Find a real number M such that
$$\left| \int_0^2 \frac{\sin it}{1 + it^2} \, \mathrm{d}t \right| \leq M.$$
   Be sure to justify your answer.

4. Is it true that for any continuous complex-valued function $w(t)$
$$\int_a^b \operatorname{Re} w(t) \, \mathrm{d}t = \operatorname{Re} \int_a^b w(t) \, \mathrm{d}t?$$
   Prove the equality or find a *specific* counterexample.

5. Evaluate
$$\int_C \operatorname{Im} z \, \mathrm{d}z$$
   where C is union of the line segment starting at 0 and ending at i and the line segment starting at i and ending at $i + 2$.

6. Is it true that for any contour C and continuous function f
$$\int_C \operatorname{Re} f(z) \, \mathrm{d}z = \operatorname{Re} \int_C f(z) \, \mathrm{d}z?$$
   Prove the equality or find a *specific* counterexample.

7. Let C be the countour from 1 to -1 following the upper half of the unit circle. Find a real number M such that
$$\left| \int_C \frac{\mathrm{d}z}{z^3 + 2} \right| \leq M.$$
   Be sure to justify your answer.