Homework 6

Reading: Chapter 4 Sections 41-47

Homework Problems:

1. Compute:
   (a) $\int_1^2 \cos(3\pi t) + it^2 - 5t \, dt$.
   (b) $\int_1^2 \frac{d}{dt} \left( \cos(3\pi it - 1) + it^2 - 5t \right) \, dt$.

2. Show that for all $m, n \in \mathbb{Z}$
   $$\int_0^{2\pi} e^{im\theta} e^{in\theta} \, d\theta = \begin{cases} 0, & m \neq n \\ 2\pi, & m = n \end{cases}$$

3. Find a real number $M$ such that
   $$\left| \int_0^2 \frac{\sin it}{1 + it^2} \, dt \right| \leq M.$$
   Be sure to justify your answer.

4. Is it true that for any continuous complex-valued function $w(t)$
   $$\int_a^b \text{Re} \, w(t) \, dt = \text{Re} \int_a^b w(t) \, dt?$$
   Prove the equality or find a specific counterexample.

5. Evaluate
   $$\int_C \text{Im} \, z \, dz$$
   where $C$ is union of the line segment starting at 0 and ending at $i$ and the line segment starting at $i$ and ending at $i + 2$.

6. Is it true that for any contour $C$ and continuous function $f$
   $$\int_C \text{Re} \, f(z) \, dz = \text{Re} \int_C f(z) \, dz?$$
   Prove the equality or find a specific counterexample.

7. Let $C$ be the contour from 1 to $-1$ following the upper half of the unit circle. Find a real number $M$ such that
   $$\left| \int_C \frac{dz}{z^3 + 2} \right| \leq M.$$
   Be sure to justify your answer.