Math 4552 – April 8, 2015 Due Wednesday, April 15, 2015

Homework 10

Reading: Chapter 6 Sections 74-81

Homework Problems:

1. Prove that the series
$$\sum_{n=1}^{\infty} \frac{z^n}{1+z^{2n}}$$
converges on both the interior and exterior of the unit circle to holomorphic functions on each region.

2. Differentiate the Maclaurin series for $1/(1-z)$ to give the Maclaurin series for
$$\frac{1}{(1-z)^2}.$$

3. Compute the following residues:

   (a)
$$\operatorname*{Res}_{z=0} \frac{e^z - 1}{z^2}.$$

   (b)
$$\operatorname*{Res}_{z=1} \frac{z}{z^2-1}.$$

4. Where are the poles of $f(z) = 1/\cos z$ and what are their orders?

5. Find all singular points of
$$f(z) = \frac{1}{z^3 - 3}$$
and compute the residue of f at each of those points.

6. Recall that when $\operatorname{Re} z > 1$ the Riemann zeta function is given by the series
$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z}$$
where $1/k^z$ is taken to mean the principal value $\text{P.V.}\,(1/k)^z = e^{-z \ln k}$.

   (a) Give the Taylor series at $z = 2$ for
$$g_k(z) = e^{-z \ln k}.$$

   (b) Notice that for $\operatorname{Re} z > 1$
$$\zeta(z) = \sum_{k=1}^{\infty} g_k(z).$$
   Give the Taylor series for $\zeta(z)$ at $z = 2$.