Math 4552
Complex Analysis

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Ohio State University

January 14, 2015

Course Info

Course Info

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office Math Tower (MW) 650

text R. V. Churchill and J. W. Brown, *Complex variables and applications*, ninth edition, McGraw-Hill, New York, 2013. ISBN-13: 978-0073383170

room Scott Laboratory (SO) N056

time MWF 12:40pm-1:35pm

webpage https://people.math.osu.edu/broaddus.9/4552/

Announcements

1. Course webpage now up.
2. Assignment 1 due Wed. 1/21

Last time

1. Defined the complex numbers (as pairs of real numbers) and how to multiply and add them
2. Listed the algebraic properties of $\mathbf{C}$ and proved a few of them
3. Saw how to compute $1/z$
4. Put the notation $x + iy$ in our context and saw that $\mathbf{R} \subset \mathbf{C}$.

Example 1

1. Put $w = 1/(-3 + 4i)$ in the form $a + bi$.
2. Show that $w(-3 + 4i) = 1$

Complex conjugation

Definition 2 (Complex conjugation)

If $z = x + iy$ then the **complex conjugate** of z is $\overline{z} = x - iy$.

Example 3

1. $\overline{-2 + 3i} = -2 - 3i$
2. $\overline{4i} = -4i$
3. $\overline{6} = 6$

Proposition 4 (Fundamental properties of conjugation)

1. $\overline{0} = 0$ *and* $\overline{1} = 1$
2. $\overline{z + w} = \overline{z} + \overline{w}$
3. $\overline{zw} = \overline{z} \cdot \overline{w}$
4. $\overline{\overline{z}} = z$

Proposition 5 (Inverse of conjugate)

1. $\overline{z^{-1}} = (\overline{z})^{-1}$ *if* $z \neq 0$.

Proof.

Suppose $z \neq 0$. Then by Existence of multiplicative inverses there is $z^{-1} \in \mathbf{C}$ such that $z^{-1}z = 1$. Hence

$$\overline{z^{-1}z} = \overline{1}$$
$$\overline{z^{-1}} \cdot \overline{z} = 1$$
$$\overline{z^{-1}} = (\overline{z})^{-1}$$

□

Example 6

- Recall from Example 1 above that $1/(-3+4i) = -\frac{3}{25} - \frac{4}{25}i$.
- Thus $1/\overline{(-3+4i)} = \overline{-\frac{3}{25} - \frac{4}{25}i} = -\frac{3}{25} + \frac{4}{25}i$

Complex division and conjugation

To write $\frac{z}{w}$ in the form $a + bi$ multiply by $\frac{\overline{w}}{\overline{w}}$

Example 7

Write $\frac{2+4i}{1-5i}$ in the form $a + bi$:

$$\begin{aligned}
\frac{2+4i}{1-5i} &= \frac{2+4i}{1-5i} \cdot \frac{\overline{1-5i}}{\overline{1-5i}} \\
&= \frac{2+4i}{1-5i} \cdot \frac{1+5i}{1+5i} \\
&= \frac{2+10i+4i-20}{1+5i-5i+25} \\
&= \frac{-18+14i}{26} \\
&= -\frac{18}{26} + \frac{14}{26}i \\
&= -\frac{9}{13} + \frac{7}{13}i
\end{aligned}$$

Redefinition of Re and Im

Definition 8 (Proper definition of Re and Im)

$$\operatorname{Re} z = \frac{z + \overline{z}}{2}, \qquad \operatorname{Im} z = \frac{z - \overline{z}}{2i}$$

Example 9

Compute $\operatorname{Im}(2 - i)$ using the definition:

$$\begin{aligned} \operatorname{Im}(2 - i) &= \frac{(2 - i) - \overline{(2 - i)}}{2i} \\ &= \frac{2 - i - (2 + i)}{2i} \\ &= \frac{-2i}{2i} \\ &= -1 \end{aligned}$$

Modulus

Lemma 10

$z \cdot \overline{z} \in \mathbf{R}^{\geq 0}$

Definition 11 (Modulus)

The **modulus** (or **absolute value**) of $z \in \mathbf{C}$ is the nonnegative real number

$$|z| = \sqrt{z \cdot \overline{z}}$$