Math 6802 – February 18, 2015 Due Wednesday, February 25, 2015

Homework 4

Homework Problems:

1. Hatcher p. 205 Problem 9
2. Hatcher p. 205 Problem 12
3. Prove Proposition 6 of Lecture 10 which asserts the following: If

$$f : A \to B$$

$$g : C \to D$$

are chain maps a then

$$f \otimes g : A \otimes C \to B \otimes D$$

is a chain map.

4. Prove Lemma 2.4 of Lecture 14 which asserts that for $u, u_1, u_2 \in C^p(X; G_1)$ and $v, v_1, v_2 \in C^q(Y; G_2)$
   (a) If u and v are cocycles then $u \times v$ is a cocycle.
   (b) Given cohomologous cocycles u_1 and u_2 and cocycle v we get $u_1 \times v$ cohomologous to $u_2 \times v$.
   (c) Given cohomologous cocycles v_1 and v_2 and cocycle u we get $u \times v_1$ cohomologous to $u \times v_2$.
5. In our proof of the Acyclic Models Theorem (Lecture 13) we carefully constructed our natural transformation $\varphi : T \to S$ giving a chain map $\varphi_X : T(X) \to S(X)$ for every object $X \in \mathcal{C}$ inducing the natural isomorphism $\Phi_X : H_0T(X) \to H_0S(X)$. Given two natural transformations $\varphi, \varphi' : T \to S$ inducing the natural transformation $\Phi : H_0T \to H_0S$. Carefully construct a natural transformation $P : T \to S$ so that for each object $X \in \mathcal{C}$ we get a chain homotopy $P_X : \varphi_X \to \varphi'_X$.
6. Recall from Lecture 14 that given $u \in H^p(X; G_1)$, $v \in H^q(Y; G_2)$ and $w \in H^r(Z; G_3)$ we have:
   (a) Associativity

$$u \times (v \times w) = (u \times v) \times w.$$

   (b) Commutativity

$$s^*(u \times v) = (-1)^{pq} v \times u.$$

   where $s : Y \times X \to X \times Y$ is the continuous map swapping factors.

   (c) Existence of units

$$u \times 1_Y = \pi_Y^*(u)$$
$$1_X \times v = \pi_X^*(v)$$

   where $1_W \in H^0(W; \mathbf{Z})$ is the cohomology class of the augmentation map $\varepsilon : C_0(W) \to \mathbf{Z}$ and $\pi_X : X \times Y \to X$ and $\pi_Y : X \times Y \to Y$ are the projections.

   Which of these statements can be made for $u \in C^p(X; G_1)$, $v \in C^q(Y; G_2)$ and $w \in C^r(Z; G_3)$? Prove them or give counterexamples.