1. Rewrite the parametric equation as \( y \) equals a function of \( x \).

\[ x = t^2 + 6, \quad y = 4t; \quad -4 \leq t \leq 4 \]

**Solution:** \( t = \frac{y}{4} \). Thus

\[ x = \left( \frac{y}{4} \right)^2 + 6 = \frac{1}{16} y^2 + 6, \]

and

\[ y = \pm \sqrt{16x - 6} \cdot 16. \]

2. Find a parametric equation of the circle centered at \((2, 4)\) with radius 2 generated clockwise.

**Solution:**

\[ x(t) = 2 + 2 \cos(\theta) \]

\[ y(t) = 4 - 2 \sin(\theta). \]

3. Consider the parametric \( x = \sqrt{t}, \quad y = 4t \).

Find \( \frac{dy}{dx} \) in terms of \( t \) and evaluate the derivative at \( t = 3 \).

**Solution:** Recall that \( \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \). Thus we need to calculate \( dx/dt \) and \( dy/dt \).

\[ \frac{dx}{dt} = \frac{1}{2} t^{-\frac{1}{2}}, \]

and

\[ \frac{dy}{dt} = 4. \]

Thus

\[ \frac{dy}{dx} = \frac{4}{\frac{1}{2} t^{-\frac{1}{2}}} = 8\sqrt{t}. \]

At \( t = 3 \),

\[ \frac{dy}{dx} = 8\sqrt{3}. \]

4. (Test Problem) Consider the following parametric curve:

\[ x = t^2, \quad y = t^3 - 12t, \quad -\infty < t < \infty \]

Find \( \frac{dy}{dx} \) in terms of \( t \). Then find all points \((x, y)\) on the curve at which there is a horizontal tangent line.

**Solution:** Using the chain rule, we can express \( \frac{dy}{dx} \) in terms of \( \frac{dy}{dt} \) and \( \frac{dx}{dt} \):

\[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 12}{2t}. \]

Points with a horizontal tangent line are points where the slope \( \frac{dy}{dx} \) is 0:

\[ \frac{3t^2 - 12}{2t} = 0 \]

\[ 3t^2 - 12 = 0 \]

\[ t^2 = 4 \]

\[ t = \pm 2. \]

So the points corresponding to \( t = \pm 2 \) will have horizontal tangent lines. Plugging in \( t \) to find \( x \) and \( y \), we get the two points. At \( t = 2 \), we have

\[ x = 4, \quad \text{and} \quad y = 2^3 - 12 \cdot 2 = -16. \]

At \( t = -2 \), we have

\[ x = 4 \quad \text{and} \quad y = (-2)^3 - 12(-2) = 16. \]

Thus the two points are \((4, 16)\) and \((4, -16)\).
5. (Test Problem) Find an equation of the line tangent to the curve at the point corresponding to the given value of \( t \).

\[
x = t^3 + t, \quad y = t^4 - t; \quad t = 1
\]

**Solution:** Recall that for parametric equations,

\[
\frac{dy}{dx}(t) = \frac{dy/dt}{dx/dt}.
\]

\[
dy/dt = 4t^3 - 1, \quad dx/dt = 3t^2 + 1.
\]

Thus

\[
\frac{dy}{dx}(t) = \frac{4t^3 - 1}{3t^2 + 1}.
\]

At the time \( t = 1 \)

\[
\frac{dy}{dx}(1) = \frac{4 \cdot 1^3 - 1}{3 \cdot 1^2 + 1} = \frac{3}{4}.
\]

At \( t = 1 \) \( x = 1^3 + 1 = 2 \) and \( y = 1^4 - 1 = 0 \).

Thus in point slope form our line is

\[
y - 0 = \frac{3}{4}(x - 2),
\]

or

\[
y = \frac{3}{4}x - \frac{3}{2}.
\]

6. (Test Question) True or False: The parametric equations \( x = -3 + \sin(3t) \), \( y = 1 + \cos(3t) \), for \( 0 \leq t < 2\pi \), generate a circle centered at \((3, 1)\) and trace the circle exactly once.

**False:** The circle will be centered at the point \((-3, 1)\) and will trace the circle 3 times.