1. Compute the dot product between $u = i - 3j + 4k$ and $v = 2i + 4j + 2k$

Solution:
\[
\begin{align*}
\mathbf{u} \cdot \mathbf{v} &= 1 \cdot 2 + 3 \cdot (-4) + 4 \cdot 2 = -2
\end{align*}
\]

2. Let $u = (5,0,15)$ and $v = (4,1,-1)$.
Calculate $\text{proj}_v u$ (the orthogonal projection of $u$ onto $v$) and $\text{scal}_v u$ the scalar component of $u$ in the direction $v$. Recall that $\text{proj}_v u = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v}$, and $\text{scal}_v u = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}$.

Solution: First let us calculate $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{v} \cdot \mathbf{v}$, and $|\mathbf{v}|$.
\[
\begin{align*}
\mathbf{u} \cdot \mathbf{v} &= 5 \cdot 4 + 0 \cdot 1 + 15 \cdot (-1) = 20 - 15 = 5 \\
\mathbf{v} \cdot \mathbf{v} &= 4 \cdot 4 + 1 \cdot 1 + (-1) \cdot (-1) = 16 + 1 + 1 = 18 \Rightarrow |\mathbf{v}| = \sqrt{18} = 3\sqrt{2}
\end{align*}
\]
Thus
\[
\begin{align*}
\text{proj}_v u &= \frac{5}{18} \mathbf{v} = \frac{5}{18} \langle 4,1,-1 \rangle = \left\langle \frac{10}{9}, \frac{5}{18}, -\frac{5}{18} \right\rangle, \\
\text{scal}_v u &= \frac{5}{3\sqrt{2}} = \frac{5\sqrt{2}}{6}.
\end{align*}
\]

3. A constant force $F = \langle 2, 3, 4 \rangle$ Newtons moves an object from the origin to the point $\langle 5, 1, 2 \rangle$ (assume our vector space is scaled in meters). What is the work done?

Solution:
\[
\text{Work} = F \cdot d = 2 \cdot 5 + 3 \cdot 1 + 4 \cdot 2 = 22 \text{Jewels}.
\]

4. A box is pulled 12m along flat ground with a force of 6N at an angle of 30° above the horizon. What is the work done?

Solution: Let us introduce a vector space to model our situation. Let the first coordinate denote the direction the box is moving in and let the second coordinate denote the vertical direction. Thus the vector representing the displacement is $\langle 12, 0 \rangle$. The vector representing force has magnitude 6 and since it’s angle is 30° above the horizon, the angle is parallel to $\langle \cos(30°), \sin(30°) \rangle = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$.

Thus
\[
\text{Work} = \langle 12, 0 \rangle \cdot \left( 6 \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \right) = 36\sqrt{3}.
\]

5. Let $a$ and $b$ be real numbers. Find all vectors $\langle 4, a, b \rangle$ orthogonal to $\langle 5, 4, -8 \rangle$.

Solution: Recall that two vectors are orthogonal when their dot product is 0. Hence we are looking for numbers $a, b$ such that
\[
0 \langle 4, a, b \rangle \cdot \langle 5, 4, -8 \rangle = 20 + 4a - 8b
\]
Equivalently
\[
a = 2b - 5.
\]
Thus any $a$ and $b$ satisfying the above relation will force the vectors to be orthogonal.