1. Find an equation for the plane passing through the point \((1, 2, 3)\) and normal to the vector \((1, 1, -1)\).
   
   Solution: As given by the above equation, the equation for the plane we are interested in is
   
   \[1(x - 1) + 1(y - 2) + (-1)(z - 3) = 0.\]
   
   After simplifying
   
   \[x + y + z = 0.\]

2. Find an equation for the plane defined by the three points \((1, 1, 1)\), \((0, 4, 0)\), and \((-2, 3, 1)\).
   
   Solution: First let us use the cross product to find a vector normal to the plane. The vector from \((1, 1, 1)\) to \((0, 4, 0)\) is
   
   \[u = \langle -1, 3, -1 \rangle\]
   
   and the vector from \((1, 1, 1)\) to \((-2, 3, 1)\) is
   
   \[v = \langle -3, 2, 0 \rangle.\]
   
   \[
u \times v = \begin{vmatrix} i & j & k \\ -1 & 3 & -1 \\ -3 & 2 & 0 \end{vmatrix} = i(3 \cdot 0 - (-1) \cdot 2) - j((-1) \cdot 0 - (-1) \cdot (-3)) + k((-1) \cdot 2 - (-3) \cdot 3) = 2i + 3j + 7k.\]
   
   Thus the equation of the plane is
   
   \[2(x - 1) + 3(y - 1) + 7(z - 1) = 0.\]
   
   After simplifying
   
   \[2x + 3y + 7z = 12.\]

3. Find an equation of the plane parallel to the plane \(Q : x - 2y - 3z = 4\) going through the pt \((2, 0, -4)\).
   
   Solution: Parallel planes have the same normal vector up to multiplication by a constant. The normal vector for the first plane is \((1, -2, -3)\). Hence we have a normal vector and a point and can form the plane:
   
   \[1(x - 2) - 2(y) - 3 + (z + 4) = 0.\]

4. Determine if the planes \(Q : 3x - 2y + z = 12\) and \(R : -x + 2y/3 - z/3 = \) are parallel, orthogonal or neither.
   
   Solution: The normal vector to \(Q\) is \(u = \langle 3, -2, 1 \rangle\) and the normal vector to \(R\) is \(v = \langle -1, 2/3, -1/3 \rangle\). Since \(u = -3v\), the vectors are parallel and hence the planes are parallel.

5. True or false: The plane passing through the point \((1, 3, 1)\) with a normal vector \(n = \langle 1, 3, -3 \rangle\) is the same as the plane passing through the point \((3, 0, 1)\) with a normal \(n = \langle -2, -6, 6 \rangle\).
   
   True: The equation of the first plane is given by
   
   \[1(x - 1) + 3(y - 3) - 3(z - 1) = 0.\]
   
   After some simplification, we reduce this equation to
   
   \[x + 3y - 3z = 7.\]
   
   The equation of the second plane is given by
   
   \[-2(x - 3) - 6(y - 0) + 6(z - 1) = 0.\]
   
   We simplify this equation as follows:
   
   \[(x - 3) + 3(y) - 3(z - 1) = 0\]
   
   \[x - 3 + 3y - 3z + 3 = 0\]
   
   \[x + 3y - 3z = 0.\]
   
   Hence they define distinct parallel planes.
6. Consider the Hyperboloid of one sheet

\[ \frac{x^2}{25} + \frac{y^2}{9} - z^2 = 1. \]

Find the intercepts with the three coordinate axis and find the equations of the \(xy\)-, \(xz\)-, \(yz\)-, traces.

**Solution:** If \(y, z = 0\), then it follows that

\[ \frac{x^2}{25} = 1. \]

In other words, \(x = \pm 5\) is the \(x\)-intercept. If \(x, z = 0\), then it follows that

\[ \frac{y^2}{9} = 1. \]

In other words, \(y = \pm 3\) is the \(y\)-intercept. If \(x, y = 0\), then it follows that

\[ -z^2 = 1. \]

In other words there are no \(z\)-intercepts.

If \(z = 0\), the \(xy\)-trace,

\[ \frac{x^2}{25} + \frac{y^2}{9} = 1, \]

is an ellipse. If \(y = 0\) the \(xz\)-trace,

\[ \frac{x^2}{25} - z^2 = 1, \]

is a hyperboloid. If \(x = 0\) the \(yz\)-trace,

\[ \frac{y^2}{9} - z^2 = 1, \]

is a hyperboloid.

Identify the following surfaces.

7. \[ \frac{x^2}{36} + \frac{y^2}{16} + \frac{z^2}{4} = 1 \]

**Solution:** As all of the variables are squared and all coefficients are positive, we have an ellipsoid.

8. \[ \frac{x}{36} - \frac{y^2}{64} + \frac{z^2}{49} = 1 \]

**Solution:** As two of the variables are squared and one is linear, and the sign of the squared terms are different, we have an hyperbolic paraboloid.

9. \[ \frac{x^2}{4} + 16y^2 - \frac{z^2}{4} = 1 \]

**Solution:** As all of the variables are squared and two coefficients are positive and one is negative, we have an hyperboloid of one sheet.

10. \[ \frac{x^2}{144} + \frac{y^2}{81} - \frac{z^2}{4} = 0 \]

**Solution:** As we have all squared variables which are equal to 0 while not all of the coefficients are the same, we have an elliptic cone.

11. \[ \frac{x^2}{25} - \frac{y^2}{25} - 4z^2 = 1 \]

**Solution:** As all of the variables are squared and two coefficients are negative and one is positive, we have an hyperboloid of one sheet.

12. \[ \frac{-x^2}{4} + 7y - \frac{z^2}{16} = 1 \]

**Solution:** As two of the variables are squared and one is linear, and the sign of the squared terms are the same, we have an elliptic paraboloid.