Instructions

- SHOW ALL WORK!!! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers may receive NO credit.

- Give EXACT answers unless asked to do otherwise.

- You do not need to simplify numerical answers such as \( \frac{5}{\sqrt{8}} - \frac{5}{\sqrt{12}} \).

- Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA’s, laptops, and cell phones are prohibited. Do not have these devices out!

- The exam duration is 55 minutes.

- The exam consists of 5 problems starting on page 2 and ending on page 6. Make sure your exam is not missing any pages before you start. Page 7 contains formulas. Page 8 may be used for extra work space.

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Problem 1

[21 pts] True or False. Give a brief explanation or example to justify your answer.

a) [3 pts] Given a solid generated by revolving a region about the line $x = 3$, if we are using the shell method to compute its volume, then we would integrate with respect to $x$.

b) [3 pts] Given a spring that obeys Hooke’s Law, the work required to stretch the spring from equilibrium to 1 cm is the same as the work required to stretch the spring from 1 cm to 2 cm.

c) [3 pts] $\int \sin^6 \theta \cos^3 \theta \, d\theta = \int (u^6 - u^8) \, du$ where $u = \sin \theta$.

d) [3 pts] $\int \frac{1}{\cos x} \, dx = \ln |\cos x| + C$

e) [3 pts] $\int \frac{x^3}{(x^2 - 4)^{3/2}} \, dx = \int \frac{\sec^3 \theta}{\tan^3 \theta} \, d\theta$, where $x = 2 \sec \theta$.

f) [3 pts] A thin wire over $0 \leq x \leq 6$ (in meters) with the density function (in $\frac{kg}{m}$) $\rho(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 4 \\ 3, & \text{if } 4 < x \leq 6 \end{cases}$, has a mass of 16 kg.

g) [3 pts] Assuming Hooke’s law is obeyed, a spring that requires 100 J of work to be stretched 4 m from its equilibrium position would have a spring constant of $k = 12.5 \, N/m$. 
Problem 2

[18 pts] Let $R$ be the region bounded by the graphs of $y = \sqrt{12x}$ and $y = 2x$.

a) [6 pts] Set up an integral with respect to $x$ that represents the area of the region $R$. DO NOT EVALUATE THE INTEGRAL.

b) [6 pts] Set up an integral with respect to $y$ that represents the area of the region $R$. DO NOT EVALUATE THE INTEGRAL.

c) [6 pts] Set up an integral that represents the volume of the solid whose base is the region $R$ and whose cross sections perpendicular to the $x$-axis are semicircles. DO NOT EVALUATE THE INTEGRAL.
Problem 3

[21 pts] Applications. **Show your work.**

a) [11 pts] An arch is to be modeled by the function $y = e^{3x} + \frac{1}{36}e^{-3x}$ on the interval $[-1, 1]$. Find the length of the arch. You do not need to simplify your final answer.

b) [10 pts] A 20 meter long rope hangs free from a ledge. The linear density of the rope is 0.3 $kg/m$. How much work is needed to pull the entire rope to the height of the ledge?
Problem 4

[20 pts] Evaluate the following integrals. Show your work.

a) [10 pts] \( \int x^8 \ln x \, dx \)

b) [10 pts] \( \int \frac{\sin (2\theta)}{\sqrt{3 + \cos (2\theta)}} \, d\theta \)
Problem 5

[20 pts] Consider the region under the graph \( y = (ax^2 + 1)^{10} \) over the interval \( 0 \leq x \leq 1 \), where \( a > 0 \) is a constant.

\[ y = (ax^2 + 1)^{10} \]

\[ 1 \quad x \]

a) [12 pts] Use the **shell method** to set up an integral (or integrals) that represents the volume of the solid generated by revolving the region about the \( y \)-axis.

b) [8 pts] Evaluate your integral(s) in part a to find the volume of the solid. Note: Your answer will possibly contain the constant \( a \).
A Few Trigonometric Identities

1) \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \)

2) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \)

3) \( \cos^2 \theta + \sin^2 \theta = 1 \)

4) \( \sec^2 \theta - \tan^2 \theta = 1 \)

5) \( \csc^2 \theta - \cot^2 \theta = 1 \)

A Few Reduction Formulas

Assume \( n \) is a positive integer.

1) \( \int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx \)

2) \( \int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx \)

3) \( \int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx, \quad n \neq 1 \)

4) \( \int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx, \quad n \neq 1 \)