1. Give an example of a non-increasing sequence with a limit.  
Solution: Consider the sequence \( a_n \) with \( a_n = \frac{1}{n} \). The sequence is non-increasing since \( a_n = \frac{1}{n} > \frac{1}{n+1} = a_{n+1} \). As \( n \to \infty \), \( \frac{1}{n} \to 0 \).

2. Give an example of a non-increasing sequence without a limit.  
Solution: Consider the sequence \( a_n \) with \( a_n = -n \). The sequence is non-increasing since \( a_n = -n > -n - 1 = a_{n+1} \). The sequence shrinks without bound and thus has no limit.

3. Give an example of a bounded sequence without a limit.  
Solution: Consider the sequence \( a_n \) with \( a_n = (-1)^n \). The sequence is bounded since all of the terms are 1 or \(-1\). The sequence has no limit since it alternates between \(-1\) and 1.

4. \( a_n = \frac{n}{\sqrt{4n^2 + 2n + 1}} \)  
Solution:  
\[
\lim_{n \to \infty} \frac{n}{\sqrt{4n^2 + 2n + 1}} = \lim_{n \to \infty} \frac{n}{\sqrt{4n^2 + 2n + 1}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{1}{\sqrt{\frac{4n^2 + 2n + 1}{n^2}}} = \lim_{n \to \infty} \frac{1}{\sqrt{4 + \frac{2}{n} + \frac{1}{n^2}}} = \lim_{n \to \infty} \frac{1}{\sqrt{4}} = \frac{1}{2}.
\]

5. \( a_n = \frac{n}{e^{n+2}} \)  
Solution: The growth rate of the numerator is polynomial which is less than the growth rate of the denominator which is exponential. Thus the limit of the sequence is 0.

6. \( a_n = \left(1 + \frac{3}{n}\right)^{3n} \)  
Solution: Let \( b_n = \ln(a_n) \) for each \( n \). If \( \lim_{n \to \infty} b_n = L \), then  
\[
\lim_{n \to \infty} a_n = e^L.
\]

\[
\lim_{n \to \infty} \ln \left[ \left(1 + \frac{3}{n}\right)^{3n}\right] = \lim_{n \to \infty} 3n \ln \left(1 + \frac{3}{n}\right) = \lim_{n \to \infty} \ln \left(1 + \frac{3}{n}\right) = \lim_{n \to \infty} \frac{1}{\frac{3n}{1 + \frac{3}{n}}} = \lim_{n \to \infty} \frac{9}{1 + \frac{3}{n}} = 9
\]

Thus \( \lim_{n \to \infty} a_n = e^9 \).

7. \( a_n = \ln(\sin(1/n)) + \ln(n) \)  
Solution:  
\[
\lim_{n \to \infty} \ln(\sin(1/n)) + \ln(n) = \lim_{n \to \infty} \ln(n \sin(1/n)) = \lim_{n \to \infty} \ln \left(\frac{\sin(1/n)}{\frac{1}{n}}\right).
\]
\[ \ln \left( \lim_{n \to \infty} \frac{\sin(1/n)}{1/n} \right) \]

L.H. \( \ln \left( \lim_{n \to \infty} \cos(1/n) \right) \)

= \ln (1) = 0

8. \( a_n = \frac{\cos(n)}{2^n} \)

**Solution:** Recall that 
\[-1 \leq \cos(n) \leq 1.\]

Thus 
\[\frac{-1}{2^n} \leq \frac{\cos(n)}{2^n} \leq \frac{1}{2^n}.\]

The growth rate of \( 2^n \) is greater than that of \( 1 \), so
\[\lim_{n \to \infty} \frac{-1}{2^n} = 0, \text{ and } \lim_{n \to \infty} \frac{1}{2^n} = 0.\]

Thus by the squeeze theorem
\[\lim_{n \to \infty} \frac{\cos(n)}{2^n} = 0.\]

9. \( a_n = \frac{4n^3 + 3}{\sqrt{n + 2n^6}} \)

**Solution:**
\[\lim_{n \to \infty} \frac{4n^3 + 3}{\sqrt{n + 2n^6}} = \lim_{n \to \infty} \frac{4n^3 + 3}{\sqrt{n} \cdot \sqrt{1 + \frac{2}{n^5}}} \]

= \[\lim_{n \to \infty} \frac{4 + \frac{3}{n^5}}{\sqrt{1 + \frac{2}{n^5}}} \]

= \[\lim_{n \to \infty} \frac{4 + \frac{3}{n^5}}{\sqrt{\frac{n + 2n^6}{n^6}}} \]

10. \( b_n = \frac{4 + \cos n}{2n} \)

**Solution:**
\[-1 \leq \cos(n) \leq 1 \]
\[3 \leq 4 + \cos(n) \leq 5 \]
\[\frac{3}{2n} \leq \frac{4 + \cos(n)}{2n} \leq \frac{5}{2n} \]

\[\lim_{n \to \infty} \frac{3}{2n} = 0 \text{ and } \lim_{n \to \infty} \frac{5}{2n} = 0.\] Thus by the squeeze theorem
\[\lim_{n \to \infty} \frac{4 + \cos(n)}{2n} = 0.\]

11. Express the sequence \( \{2n + 1\}_{n=1}^{\infty} \) as an equivalent sequence of the form \( \{b_m\}_{m=1}^{\infty} \)

**Solution:** This requires only a careful reindexing. \( n = m - 3 \) Thus \( a_n = 2(n - 3) + 1 = 2m - 5 \). Thus the choice of \( b_m = 2m - 5 \) provides the equivalent sequence.

12. Given \( a_1 = 0 \) and \( a_{n+1} = \frac{6}{3 - a_n} \) for \( n \geq 1 \), the first four terms of the sequence \( \{a_n\}_{n=1}^{\infty} \) are nonnegative

**False:**
\[a_2 = \frac{6}{3 - a_1} = \frac{6}{3 - 0} = 2\]
\[a_3 = \frac{6}{3 - a_2} = \frac{6}{3 - 2} = 6\]
\[a_4 = \frac{6}{3 - a_3} = \frac{6}{3 - 6} = -2.\]