1. \[\sum_{k=0}^{9} \left( -\frac{3}{4} \right)^k \]

**Solution:** In this case, we have a finite geometric sum with \(a = 1\), \(r = \frac{-3}{4}\), and \(n = 10\). Thus
\[\sum_{k=0}^{9} \left( -\frac{3}{4} \right)^k = 1 \cdot \frac{1 - \left( -\frac{3}{4} \right)^{10}}{1 - \left( -\frac{3}{4} \right)}.
\]

2. \[\sum_{k=1}^{5} \left( -\frac{5}{2} \right)^k \]

**Solution:** After reindexing so that our first term corresponds to \(k = 0\), we have
\[\sum_{k=1}^{5} \left( -\frac{5}{2} \right)^k = \sum_{k=0}^{4} \left( -\frac{5}{2} \right)^{k+1} = \sum_{k=0}^{4} \left( -\frac{5}{2} \right) \left( -\frac{5}{2} \right)^k.
\]
Thus we have a finite geometric sum with \(a = \left( \frac{-5}{2} \right)\), \(r = \left( \frac{-5}{2} \right)\), and \(n = 5\). So
\[\sum_{k=0}^{4} \left( -\frac{5}{2} \right) \left( -\frac{5}{2} \right)^k = \left( -\frac{5}{2} \right) \frac{1 - \left( -\frac{5}{2} \right)^5}{1 - \left( -\frac{5}{2} \right)}.
\]

3. \[\sum_{k=3}^{\infty} \frac{3 \cdot 4^k}{7^k} \]

**Solution:** Write the given series as
\[\sum_{k=3}^{\infty} \frac{3 \cdot 4^k}{7^k} = 3 \sum_{k=3}^{\infty} \left( \frac{4}{7} \right)^k = 3 \left( \frac{4}{7} \right)^3 + 3 \left( \frac{4}{7} \right)^7 + 3 \left( \frac{4}{7} \right)^5 + \ldots
\]

This is a geometric series with first term \(a = 3 \left( \frac{4}{7} \right)^3\) and ratio \(r = \frac{4}{7}\). Since \(|r| < 1\), the series converges, and in particular it converges to
\[S = \frac{a}{1 - r} = \frac{3 \left( \frac{4}{7} \right)^3}{1 - \frac{4}{7}} = \frac{64}{49}.
\]

4. \[\sum_{k=0}^{\infty} \left( \frac{1}{4} \right)^{5^7 - k} \]

**Solution:** Rewrite the given series as follows
\[\sum_{k=0}^{\infty} \left( \frac{1}{4} \right)^{5^7 - k} = \sum_{k=0}^{\infty} \left( \frac{1}{4} \right)^{k} \left( \frac{1}{5} \right)^k = \sum_{k=0}^{\infty} 5^7 \left( \frac{1}{20} \right)^k.
\]
Thus this is a geometric series with \(a = 5^7\) and \(r = \frac{1}{20}\). Since \(|r| < 1\), the series converges to
\[S = \frac{a}{1 - r} = \frac{5^7}{1 - \frac{1}{20}}.
\]

5. \[\sum_{k=0}^{\infty} (1.01)^k \]

**Solution:** By inspection one sees that the series is geometric with ratio \(r = |1.01| > 1\). Thus the series diverges.
6. Write the repeating decimal 
0.1\overline{2} = 0.1212121212\ldots first as a geometric series and then as a fraction.

**Solution:** First we need to express 0.1\overline{2} as a geometric series.

\[
0.1\overline{2} = 0.12 + 0.0012 + 0.000012 + \ldots = 0.12 + \frac{1}{100} + \frac{1}{100^2} + \ldots = \sum_{k=0}^{\infty} 0.12 \cdot \frac{1}{100^k}.
\]

This is a geometric series with first term \(a = 0.12\) and ratio \(r = \frac{1}{100}\). Since \(|r| < 1\), the series converges to

\[
S = \frac{a}{1 - r} = \frac{12}{1 - \frac{1}{100}} = \frac{12}{99} = \frac{4}{33}.
\]

7. \[
\sum_{k=1}^{\infty} \ln \left( \frac{k+1}{k} \right)
\]

**Solution:** After making the observation that \(\ln \left( \frac{k+1}{k} \right) = \ln(k+1) - \ln(k)\), the problem solves like 10.

8. \[
\sum_{k=1}^{\infty} \frac{1}{16k^2 + 8k - 3}
\]

**Solution:** A priori, the series appears to be unrelated to telescoping techniques. However, a partial fraction decomposition of the fraction at hand will yield the necessary telescoping structure. The denominator of the fraction \(\frac{1}{16k^2 + 8k - 3}\) factors into \((4k + 3)(4k - 1)\). Thus there exists \(A\) and \(B\) such that

\[
\frac{1}{16k^2 + 8k - 3} = \frac{A}{4k + 3} + \frac{B}{4k - 1}.
\]

After multiplying through by \(16k^2 + 8k - 3\), we have

\[
1 = A(4k - 1) + B(4k + 3).
\]

Plugging in \(k = 1/4\) into the equation gives us that

\[
1 = A(1 - 1) + B(1 + 3),
\]

or that \(B = \frac{1}{4}\), and plugging in \(k = -3/4\) into the equation gives us that

\[
1 = A((-3 - 1) + B(-3 + 3),
\]

or that \(A = -\frac{1}{4}\). Since \(4k + 3 = 4(k + 1) - 1\), our series becomes

\[
\sum_{k=0}^{\infty} \frac{1}{16k^2 + 8k - 3} = \sum_{k=0}^{\infty} \frac{1}{4k - 1} - \frac{1}{4k + 3}.
\]

Let

\[
S_n = \sum_{k=0}^{n} \frac{1}{4k - 1} - \frac{1}{4(k + 1) - 1}.
\]

Then

\[
S_n = \left(\frac{\frac{1}{4} \cdot (n - 1) - \frac{1}{4} \cdot n}{4 \cdot (n - 1) - 1 - 4 \cdot n - 1}\right) + \left(\frac{\frac{1}{4} \cdot (n - 1) - \frac{1}{4} \cdot n}{4 \cdot (n - 1) - 1 - 4 \cdot n - 1}\right) + \ldots + \left(\frac{\frac{1}{4} \cdot (n - 1) - \frac{1}{4} \cdot n}{4 \cdot (n - 1) - 1 - 4 \cdot n - 1}\right) \rightarrow \frac{1}{4} \cdot (1 - 1) - \frac{1}{4} \cdot (n + 1) - 1.
\]

Thus

\[
\sum_{k=0}^{\infty} \frac{1}{16k^2 + 8k - 3} = \lim_{n \to \infty} S_n = \frac{1}{12}.
\]
9. (TestProblem) Determine if the series converges or diverges. If it converges, find its value.

\[
\sum_{k=1}^{\infty} \frac{5^k}{2^{3k}}
\]

**Solution:** Write the given series as

\[
\sum_{k=1}^{\infty} \frac{5^k}{2^{3k}} = \sum_{k=1}^{\infty} \frac{5^k}{(2^3)^k} = \frac{5}{8} + \left(\frac{5}{8}\right)^2 + \left(\frac{5}{8}\right)^3 + \ldots
\]

This is a geometric series with first term \(a = \frac{5}{8}\) and ratio \(r = \frac{5}{8}\). Since \(|r| < 1\), the series converges to

\[
S = \frac{a}{1 - r} = \frac{\frac{5}{8}}{1 - \frac{5}{8}} = \frac{5}{3}.
\]

10. (TestProblem) Determine if the series converges or diverges. If it converges, find its value.

\[
\sum_{k=1}^{\infty} (\ln(k + 2) - \ln(k))
\]

**Solution:**

\[
S_n = \sum_{k=1}^{n} (\ln(k + 2) - \ln(k))
\]  
\[
= (\ln 3 - \ln 1) + (\ln 4 - \ln 2) + \ldots + (\ln(n + 1) - \ln(n - 1)) + (\ln(n + 2) - \ln(n)).
\]

Examining the middle terms, we find that this is a telescoping series, where all but two terms at the beginning and two at the end cancel:

\[
S_n = (\ln 3 - \ln 1) + (\ln 4 - \ln 2) + \ldots + (\ln(n + 1) - \ln(n)) + (\ln(n + 2) - \ln(n)).
\]

11. Use the properties of infinite series to evaluate the following series.

\[
\sum_{k=0}^{\infty} \left[ \frac{2}{5} \left( \frac{2}{7} \right)^k + \frac{1}{6} \left( \frac{3}{4} \right)^{k-1} \right]
\]

**Solution:** Since \(|2/7| < 1\) and \(|3/4| < 1\),

\[
\sum_{k=0}^{\infty} \left[ \frac{2}{5} \left( \frac{2}{7} \right)^k + \frac{1}{6} \left( \frac{3}{4} \right)^{k-1} \right]
\]

\[
= \frac{2}{5} \sum_{k=0}^{\infty} \left( \frac{2}{7} \right)^k + \frac{1}{6} \cdot 4 \sum_{k=0}^{\infty} \left( \frac{3}{4} \right)^k
\]

\[
= \frac{2}{5} \cdot \frac{1}{1 - \frac{2}{7}} + \frac{1}{6} \cdot \frac{4}{1 - \frac{3}{4}}.
\]

12. Evaluate the geometric series or show that it diverges.

\[
\sum_{k=2}^{\infty} (-3)^k 8^{2-k}
\]

**Solution #1:** As we are told the series is geometric we only need to calculate \(a\) and \(r\) to find the value of the series. \(a\) is equal to the first term in the series. In this case

\[
(-3)^2 \cdot 8^{2-2} = 9.
\]
$r$ is equal to the ratio of consecutive terms. In this case
\[ r = \frac{(-3)^{k+1} 8^{2-(k+1)}}{(-3)^k 8^{2-k}} = (-3)(8)^{-1} = -\frac{3}{8}. \]
Since $|r| < 1$ the series converges and is equal to
\[ \frac{a}{1-r} = \frac{9}{1-\left(-\frac{3}{8}\right)} = \frac{72}{11}. \]

**Solution #2:**
\[ \sum_{k=2}^{\infty} (-3)^k 8^{2-k} = \sum_{k=2}^{\infty} (-3)^k 8^2 8^{-k} \]
\[ = \sum_{k=2}^{\infty} 64 \cdot \left(\frac{-3}{8}\right)^k \]
\[ = \sum_{k=0}^{\infty} 64 \cdot \left(\frac{-3}{8}\right)^{k+2} \]
\[ = \sum_{k=0}^{\infty} 64 \cdot \left(\frac{-3}{8}\right)^k \cdot \left(\frac{-3}{8}\right)^2 \]
\[ = \sum_{k=0}^{\infty} 9 \cdot \left(\frac{-3}{8}\right)^k. \]
This is a geometric series with $a = 9$ and $r = -\frac{3}{8}$. Since $|r| < 1$ the series converges and is equal to
\[ \frac{a}{1-r} = \frac{9}{1-\left(-\frac{3}{8}\right)} = \frac{72}{11}. \]

13. (True or False) \( \sum_{k=1}^{\infty} \frac{2^k}{5^{k+1}} = \frac{1}{3} \)

**False:**
\[ \sum_{k=0}^{\infty} \frac{2^k}{5^{k+2}} = \frac{2}{25} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^k \]
\[ = \frac{2}{25} \cdot \frac{1}{1-\frac{2}{5}} = \frac{2}{15}. \]