Use the divergence test to determine whether the following sequence diverge or state that the test is inconclusive.

1. \[ \sum_{k=2}^{\infty} \frac{k}{\ln(k)} \]

**Solution:**

\[
\lim_{k \to \infty} \frac{k}{\ln(k)} = \lim_{k \to \infty} \frac{1}{\frac{1}{k}} = \infty.
\]

The limit of the terms of our series is not 0, hence by the divergence test the series diverges.

2. \[ \sum_{k=1}^{\infty} \frac{k^2}{2k^2 + k} \]

**Solution:**

\[
\lim_{k \to \infty} \frac{k^2}{2k^2 + k} = \lim_{k \to \infty} \frac{1}{2 + \frac{1}{k}} = \frac{1}{2}.
\]

The limit of the terms of our series is not 0, hence by the divergence test the series diverges.

3. \[ \sum_{k=1}^{\infty} k^\frac{1}{k} \]

**Solution:**

\[
\lim_{k \to \infty} k^\frac{1}{k} = \lim_{k \to \infty} e^{\ln(k^\frac{1}{k})} = e^{\lim_{k \to \infty} \frac{1}{k} \ln k} = e^{\lim_{k \to \infty} \frac{\ln k}{k}} = e^{0} = 1 \neq 0.
\]

The limit of the terms of our series is not 0, hence by the divergence test the series diverges.

4. \[ \sum_{k=1}^{\infty} \frac{k^5}{k!} \]

**Solution:**

The growth rate of \(k^5\) is greater than that of \(5^k\). So \(\lim_{k \to \infty} \frac{5^k}{k!} = 0\). Thus the divergence test is inconclusive.

5. Determine whether the following series converge or diverge. State any test that you use.

\[ \sum_{k=0}^{\infty} \sin\left(\frac{\pi k}{3k+2}\right) \]

**Solution:** I will use the test for divergence. \(^a\)

\[
\lim_{k \to \infty} \sin\left(\frac{\pi k}{3k+2}\right) = \sin\left(\lim_{k \to \infty} \frac{\pi k}{3k+2}\right) = \sin\left(\lim_{k \to \infty} \frac{\pi}{3 + \frac{2}{k}}\right) = \sin\left(\frac{\pi}{3}\right) = \sqrt{3} \neq 0.
\]

The limit of the terms is not 0. Hence by the test for divergence the series diverges. \(^a\)

6. (True or False) If \(\lim_{k \to \infty} a_k = 1\) then \(\sum_{k=0}^{\infty} a_k\) converges.

**False:** The divergence test states that if \(\lim_{k \to \infty} a_k \neq 0\), then \(\sum_{k=0}^{\infty} a_k\) diverges.