Problem 1

[18 pts] True or False. Give a brief explanation or example to justify your answer.

a) [3 pts] If \( \lim_{k \to \infty} a_k = 1 \) then \( \sum_{k=0}^{\infty} a_k \) converges.

b) [3 pts] \( \sum_{k=2}^{\infty} \frac{4}{k^2 - 1} = 4 \)

c) [3 pts] \( \int \frac{x^4}{(x^2 + 3)^{5/2}} \, dx = \int \frac{\tan^4 \theta}{\sec^3 \theta} \, d\theta \) where \( x = \sqrt{3} \tan \theta \).

d) [3 pts] The parametric equations \( x = -3 \sin(3t), y = 1 + \cos(3t) \), for \( 0 \leq t < 2\pi \), generate a circle centered at (3, 1) and trace the circle exactly once.

e) [3 pts] \( \sum_{k=1}^{\infty} \frac{2^k}{5^{k+1}} = \frac{1}{3} \)

f) [3 pts] Given \( a_1 = 0 \) and \( a_{n+1} = \frac{6}{3 - a_n} \) for \( n \geq 1 \), the first four terms of the sequence \( \{a_n\}_{n=1}^{\infty} \) are nonnegative
Problem 2

[22 pts] Determine whether the following improper integral converges or diverges. If it converges, find its value.

\[ \int_{2}^{\infty} \frac{-2x + 6}{x(x^2 + 2x - 3)} \, dx. \]
Problem 3

[18 pts] Power Series.

a) [12 pts] Find the radius and interval of convergence for the power series.

\[ \sum_{k=0}^{\infty} \frac{k^2(x + 5)^k}{3^{2k}} \]

b) [6 pts] Find the function that is represented by the series.

\[ \sum_{k=0}^{\infty} \frac{4^kx^{k+1}}{k + 1} \]
Problem 4

[20 pts] Taylor Series. Find the first four nonzero terms of the Taylor series for the given function centered at \( a \).

a) [10 pts] \( f(x) = \frac{1}{(1 + x)^2}, \quad a = 1 \)

b) [10 pts] \( g(x) = x^3 - 5x + 2, \quad a = -1 \)
Problem 5

[22 pts] Parametric and Polar equations.

a) [10 pts] Find an equation of the line tangent to the curve at the point corresponding to the given value of $t$.

\[ x = t^3 + t, \quad y = t^4 - t; \quad t = 1 \]

b) [12 pts] Find the area of the region bounded by the polar curve $r = 2 + \sin \theta$. 