Basic Logic Identities and Implications: formal versions of things you can use in proofs.

Logical Equivalences

Law of Double Negation

$$\neg(\neg A) \equiv A$$

DeMorgan's Laws

$$\neg(A \wedge B) \equiv (\neg A) \vee (\neg B)$$

$$\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$$

Negation of Conditional

$$\neg(A \Rightarrow B) \equiv A \wedge (\neg B)$$

Negation of Biconditional

$$\neg(A \Leftrightarrow B) \equiv (A \wedge (\neg B)) \vee ((\neg A) \wedge B)$$

Generalized DeMorgan Laws

$$\neg(\forall x)A \equiv (\exists x)(\neg A)$$

$$\neg(\forall x \in D)A \equiv (\exists x \in D)(\neg A)$$

$$\neg(\exists x)A \equiv (\forall x)(\neg A)$$

$$\neg(\exists x \in D)A \equiv (\exists x \in D)(\neg A)$$

Rewriting a Conditional or Biconditional

$$A \Rightarrow B \equiv (\neg A) \vee B$$

$$A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$$

Distributive Laws

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

Contrapositive

$$A \Rightarrow B \equiv (\neg B) \Rightarrow (\neg A)$$

Commutativity

$$A \wedge B \equiv B \wedge A$$

$$A \vee B \equiv B \vee A$$

Associativity

$$A \wedge (B \wedge C) \equiv (A \wedge B) \wedge C$$

$$A \vee (B \vee C) \equiv (A \vee B) \vee C$$

Tautologies

Identity

$$A \Leftrightarrow A$$

$$A \Rightarrow A$$

Law of Excluded Middle

$$A \vee (\neg A)$$

Logical Implications

Modus Ponens

$$A \Rightarrow B, A \models B$$

Modus Tolens

$$A \Rightarrow B, \neg B \models \neg A$$

Disjunctive Syllogism

$$A \vee B, \neg A \models B$$

$$A \vee B, \neg B \models A$$

Cases

$$A \vee B, A \Rightarrow C, B \Rightarrow C \models C$$

Substitution

$$A \Leftrightarrow A', \neg A \models \neg A'$$

$$A \Leftrightarrow A', B \Leftrightarrow B', A \wedge B \models A' \wedge B'$$

$$A \Leftrightarrow A', B \Leftrightarrow B', A \vee B \models A' \vee B'$$

$$A \Leftrightarrow A', B \Leftrightarrow B', A \Rightarrow B \models A' \Rightarrow B'$$

$$A \Leftrightarrow A', B \Leftrightarrow B', A \Leftrightarrow B \models A' \Leftrightarrow B'$$

Stronger Versions of Substitution

$$A' \Rightarrow A, \neg A \models \neg A'$$

$$A \Rightarrow A', B \Rightarrow B', A \wedge B \models A' \wedge B'$$

$$A \Rightarrow A', B \Rightarrow B', A \vee B \models A' \vee B'$$

$$A' \Rightarrow A, B \Rightarrow B', A \Rightarrow B \models A' \Rightarrow B'$$