Pop Quiz!

May 29, 2012

1. What is a vector?

   We will say "an object with magnitude and direction," although with specifying what we mean by "object," "magnitude," or "direction," this is meaningless.

   A better answer is "an n-tuple of real numbers."  

2. Let \( f : \mathbb{R}^2 \to \mathbb{R} \). If the surface \( S \) is the graph of \( f \), what exactly do we mean? i.e., \( S = \{(x, y, z) \mid ??\} \).

   \[ S = \{ (x, y, z) \mid z = f(x, y) \} . \]

3. What is meant by saying the surface \( T \) is the level surface of a function \( g \)? i.e., what is the definition of level surface.

   \( \text{(There is } \alpha \text{ for which) } T = \{ (x, y, z) \mid \alpha \} . \)
4. What is the difference between a graph of a function and a level surface?

A level surface is a subset of the points in the domain of a function. A graph is the collection of every \((x, y)\) with \(x\) in the domain and \(y = f(x)\).

5. Let \(v = (150, 120)\) and \(u = (2, 0)\). What is \(\text{comp}_u v\)? Let \(w = (3, 4, 100)\) and \(x = (1, 1, 0)\). What is \(\text{comp}_x w\)?

\[
\text{comp}_u v = 150, \quad \text{since} \quad \frac{u}{|u|} = \left(\frac{1}{14\sqrt{2}}, \frac{12}{14\sqrt{2}}\right) = \frac{7}{14}\sqrt{2}, \quad y = f(x).
\]

\[
\text{comp}_x w = (3, 4, 100) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = \frac{7}{\sqrt{2}}
\]

6. Have we ever actually said what it means for a function \(f : \mathbb{R}^2 \to \mathbb{R}\) to be differentiable?

No, we have not.

We have defined partial derivatives, directional derivatives, and gradients.

But derivatives are defined using elementary ideas from linear algebra, which we don't cover.