Question 1: Determine whether or not the following statements are true and give an explanation or a counterexample.

(a) The derivative \( \frac{d}{dx} (10^5) \) equals \( 5 \cdot 10^4 \).

(b) The slope of a line tangent to \( f(x) = e^x \) is never 0.

(c) \( \frac{d}{dx} (4e^x) = 4xe^{x-1} \).

(d) \( \frac{d}{dx} (e^x) = xe^{x-1} \).

(e) The \( n \)th derivative \( \frac{d^n}{dx^n} (5x^3 + 2x + 5) \) equals 0 for any integer \( n \geq 3 \).
Question 2: Assume $f$ is a differentiable function whose graph passes through the point $(1, 4)$. If $g(x) = f(x^2)$ and the tangent line to the graph of $f$ at $(1, 4)$ is $y = 3x + 1$, determine each of the following:

(a) $g(1)$

(b) $g'(x)$

(c) $g'(1)$

(d) Find the equation of the line tangent to the graph of $g$ when $x = 1$. 
Question 3. A swim coach teaches athletes how to swim freestyle while keeping their hips raised. Let $q$ represent the number of swimmers that learn the technique, and let $h$ represent the number of hours of individual training the swim coach provides in which swimmers focus on their hips. The swim team’s production function is $q = h^{1/2}$. The swim coach is paid $150 per hour of swim practice.

(a) What is the cost function $C(q)$? What is the marginal cost function $MC(q)$?

(b) Now suppose the coach has learned a new method to teach swimmers how to swim with their hips raised. His new team production function is $q = h^{3/4}$. What is the cost function, $C(q)$? What is the marginal cost function, $MC(q)$?

(c) Compare the marginal cost functions in parts (a) and (b)? (Compare here means do something mathematical to compare the two functions, and then comment.) How does the coach’s new technique affect the swim team?