Basic skills required to work through the problems:

• calculating volumes of simple solids;
• applying Pythagoras’ theorem;
• write down proportionality relations between the sides of similar triangles, and trigonometric relations between the sides of a right triangle;
• writing down a mathematical expression for revenue in terms of price and quantity of demand for a product.

Learning Goals: After completing this workshop, you should be able to:

• solve related rates problems.

1. A circular swimming pool with vertical sides is being filled from a fire hose at rate of 5 cubic feet per second. If the pool is 40 feet across, how fast is the water level increasing when the pool is one third full?

2. Read the following problem and answer the questions below.

“Ship A is 400 miles directly south of Tahiti and is sailing south at 20 miles/hr. Ship B is 300 miles east of Tahiti and is sailing west at 15 miles/hr. At what rate is the distance between the ships changing?”

(a) UNDERSTAND THE PROBLEM: What is the problem asking? Choose the correct answer from the ones below.

(a.1) The problem asks to find the rate of change of the distance of ship A from Tahiti at a certain moment in time.
(a.2) The problem asks to find the rate of change of the distance of ship B from Tahiti at a certain moment in time.
(a.3) The problem asks to find the rate of change of the distance of ship A from ship B at a certain moment in time.
(a.4) The problem asks to find the rate of change of the distance between the ships at any time.

(b) UNDERSTAND THE PROBLEM: Assign names to variables. For example, suppose $D$ is the distance between the ships, $x$ is the distance of ship A from Tahiti, $y$ is the distance of ship B from Tahiti. Draw a picture if necessary. As you determined above, you need to find the rate of change of $D$ at a specific time when $x = ...$, $y = ...$, $\frac{dx}{dt} = ...$, $\frac{dy}{dt} = ...$. What are the missing values in this sentence?

(c) THINK OF A PLAN: Which of the following actions best describes a strategy to solve the given problem? Choose the correct answer from the ones below.

(c.1) Finding the derivative of $D$ with respect to $x$.
(c.2) Finding the derivative of $D$ with respect to $y$.
(c.3) Finding the derivative of $D$ with respect to time.

(d) THINK OF A PLAN: Find an expression relating $D$ to $x$ and $y$.
(e) THINK OF A PLAN: Which of the distances defined in part (b) changes with time?
(f) CARRY OUT YOUR PLAN: Use all the information collected in parts (a) - (e) and solve the given problem.

(g) REFLECT ON WHAT YOU ARE DOING: What units is your answer expressed in? Make sure that your calculation is dimensionally consistent.
3. A manufacturer of precision weighing scales has determined that when the price of a scale is \( p \) (in dollars), it will sell a number \( q \) of scales per month that satisfies the relation \( 10p + 5q + qp^2 = 100 \). Due to inflation and to changing labour costs, both \( p \) an \( q \) depend on time. Find the rate at which the number of scales sold is changing when the price of a scale is \( p = 5 \), and the price is increasing at a rate of $2 per month. You do not need to simplify your answer in this question.

4. A right circular cone is turned upside-down (vertex is at the bottom); the height of cone is 10 cm and the diameter of the top is also 10 cm. The cone is filled half-way (by depth), and water is draining out. The depth of the water is dropping at a rate of 4 cm per second. How fast is the volume of the water changing at this time? You may use the fact that the volume of a cone with radius \( r \) and height \( h \) is \( \pi r^2 h / 3 \).

5. A circular ferris wheel with radius 10 metres is revolving at the rate of 10 radians per minute. How fast is a passenger on the wheel rising when the passenger is 6 metres higher than the centre of the wheel and is rising? Include units with your answer.

6. The price \( p \) in $ for a product and the quantity \( q \) of demand (that is, the quantity of units sold) for a product are related by the equation \( p^2 + 2q^2 = 1100 \). If the price is increasing at a rate of $2 per month when the price is $30, find the rate of change of the revenue \( R \) in dollars per month.