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Chapter 15
The Stone-Čech Compactification

15.1 Compactifications and Tychonoff Spaces

Definition (Compactifications and Isomorphic Compactifications). Recall that a compactification of a space $X$ is a compact Hausdorff space $Y$ such that there exists an embedding $\varphi: X \hookrightarrow Y$ that $\varphi[X] = Y$. Thus, a compactification in the strictest sense is a tuple $(Y, \varphi: X \hookrightarrow Y)$. Two compactifications $(Y, \varphi)$ and $(Y', \psi)$ are said to be equivalent (more categorically, isomorphic compactifications) if there is a homeomorphism $h: Y \approx Y'$ such making the following diagram commute:

\[
\begin{array}{ccc}
X & \xrightarrow{\varphi} & Y \\
\downarrow{\psi} & & \downarrow{h} \\
Y' & \xleftarrow{\varphi'} & Y \\
\end{array}
\]

For simplicity, we often just assume compactifications contain $X$ as a subspace so we need not concern ourselves with somewhat pedantic set-theoretic squabbles. Warning. Munkres assumes that the compactification is a Hausdorff space though a more general definition would not require this condition.

Remark. There might be a nice categorical way of phrasing this so let us review some stuff. Let $\mathcal{C}$ be the category whose objects are embeddings of topological spaces in compact spaces and whose arrows are pairs of continuous functions. That is, if we have two objects $f: X \rightarrow Z$ and $f': Y \rightarrow Z'$ where $Z$ and $Z'$ are compact, a morphism $f \rightarrow f'$ is defined to be a pair $(\varphi, \psi)$ of continuous maps making the following diagram commute:

\[
\begin{array}{ccc}
X & \xrightarrow{f} & Z \\
\downarrow{\varphi} & & \downarrow{\psi} \\
Y & \xrightarrow{f'} & Z' \\
\end{array}
\]

Let us first cover a simple result.

15.1.1 Normal Hausdorff Spaces are Tychonoff.

Proposition 37. Every normal Hausdorff space is Tychonoff.

Proof. Let $X$ be normal and Hasudorff; we need only show $X$ is completely regular. Let $A$ be closed in $X$ and $x \in X \setminus A$. Then $A$ and $\{x\}$ are closed. Hence, by Urysohn’s lemma, there exists a continuous function $f: X \rightarrow [0,1]$ such that $f[A] = \{0\}$ and $f[\{x\}] = 1$. ■
15.1.2 Induced Compactifications.

Lemma 20. Let $X$ be a topological space and suppose $h: X \to Z$ is an embedding of $X$ in a compact Hausdorff space $Z$. Then there exists a corresponding compactification $(Y, \varphi)$ of $X$ and an embedding $H: Y \to Z$ such that TFDC:

$$
\begin{array}{ccc}
X & \xrightarrow{\varphi} & Y \\
\downarrow{h} & & \downarrow{H} \\
Z & & 
\end{array}
$$

Any such compactification making the above diagram commute is unique up to homeomorphism.

We call $Y$ the compactification induced by the embedding $h$.

Proof. We shall give the proof in the case that $X$ is a subspace of $Y$, WLOG. Given $h$, let $X_0 = h[X]$ and $Y_0 = \overline{X_0}$. Then $Y_0$ is a compact Hausdorff space and $\overline{X_0} = Y_0$; therefore $Y_0$ is a compactification of $X_0$.

We now construct a space $Y$ containing $X$ such that the pair $(X, Y)$ is homeomorphic to the pair $(X_0, Y_0)$. Choose a set $A$ disjoint from $X$ that is in bijective correspondence with the set $Y_0 \setminus X_0$ under some map $k: A \to Y_0 \setminus X_0$ and let $Y = X \cup A$. Define a bijective correspondence $H: Y \to Y_0$ by the rule

$$
H(y) = \begin{cases} 
H(y) = h(y) & \text{for } y \in X \\
H(y) = k(y) & \text{for } y \in A.
\end{cases}
$$

Then topologize $Y$ by declaring $U \subseteq X$ to be open in $Y$ iff $H[U]$ is open in $Y_0$. Then $H$ is automatically a homeomorphism and the space $X$ is a subspace of $Y$, in fact, because $H$ equals the homeomorphism $H$ when restricted to the subspace $X$ of $Y$. By expanding the range of $H$, we obtained the required embedding of $Y$ into $Z$.

Now suppose $Y_i$ is a compactification of $X$ and that $H_i: Y_i \to Z$ is an embedding extending $h$ for $i = 1, 2$. Now, $H_i$ maps $X$ onto $h[X] = X_0$. Because $H_i$ is continuous, it must map $Y_i$ onto $\overline{X_0}$; because $X_0 \subseteq H_i[Y_i]$ and $H_i[Y_i]$ is closed (being compact), it contains $\overline{X_0}$. Hence, $H_i[Y_i] = \overline{X_0}$ and $H_2^{-1} \circ H_1$ defines a homeomorphism of $Y_1$ with $Y_2$ inducing the identity on $X$. ■

Definition (Induced Compactification). As we mentioned, the compactification above is called the induced compactification.

In general, there are many different ways of compactifying a given space. Consider for instance the following compactifications of the open interval $X = (0, 1)$.

Example 21. Take $S^1 \subseteq \mathbb{R}^2$ and $h: (0, 1) \to S^1$ the map $h(t) = (\cos(2\pi t), \sin(2\pi t))$. The compactification induced by the embedding $h$ is equivalent to the one-point compactification of $X$.

Example 22. Let $Y = [0, 1]$. Then $Y$ is a compactification of $X$; it is obtained by “adding one point at each end of $(0, 1)$” in the obvious sense.

Example 23. Consider the square $[-1, 1]^2$ in $\mathbb{R}^2$ and let $h: (0, 1) \to [-1, 1]^2$ be the map $h(x) = (x, \sin(1/x))$. The space $Y_0 = h[X]$ is the topologist’s sine curve. The embedding $h$ gives rise to a compactification of $(0, 1)$ quite different from the other two: It is obtained by adding one point at the right-hand end of $(0, 1)$, and an entire line segment of points at the left-hand end.

15.1.3 The Existence of a Hausdorff Compactification is Equivalent to Being Tychonoff.

Proposition 38. A space $X$ has a Hausdorff compactification iff $X$ is Tychonoff.

Proof. $(\implies)$ If $X$ has a Hausdorff compactification $Y$, then $X$ must be completely regular since every compact Hausdorff space is normal by Theorem 1.64 and since every normal Hausdorff space is completely regular by Proposition 36 and complete regularity is hereditary, $X$ is completely regular. $(\impliedby)$ Since $X$ is Tychonoff, it can be embedded in $[0, 1]^J$ for some $J$ by Theorem 1.71 which is Compact by Tychonoff’s Theorem. By Lemma 21, it is clear that $X$ has a Hausdorff compactification since Hausdorffness is preserved under product and is hereditary. ■
A Basic Problem in the Study of Compactifications.

A basic problem in the study of compactifications is the following: If $Y$ is a compactification of $X$, under what conditions can a continuous real-valued function $f$ defined on $X$ be extended continuously to $Y$? The function $f$ will have to be bounded if it is extendable, since its extension will carry the compact space $Y$ into $\mathbb{R}$ and, thus, be bounded. But this is not enough in general.

**Example 24.** Let $X = (0, 1)$. Consider the one-point compactification of $X$ induced by the embedding $h: (0, 1) \to S^1$ by $h(t) = (\cos(2\pi t), \sin(2\pi t))$. That is, $X_0 = [0, 1]$ and $Y = X/(0 \sim 1)$, so that $Y$ is, in fact, homeomorphic to $S^1$. A bounded continuous function $f: (0, 1) \to \mathbb{R}$ is extendable to this compactification iff $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$ exist and $f(0^+) = f(0^-)$. For "the two-point compactification" of $X$ considered in the second example above. In this case, the function $f$ is extendable iff these limits simply exist.

Finally, consider the last example above. It is easy to see that $f$ is extendable if both the above limits exist. But the function $f(x) = \sin(1/x)$ is also extendable to this compactification: Let $H$ be the embedding of $Y$ in $\mathbb{R}^2$ that equals $h$ on the subspace $X$. Then the composite map

$$ Y \xrightarrow{H} \mathbb{R} \times \mathbb{R} \xrightarrow{\pi_2} \mathbb{R} $$

is the desired extension of $f$. For if $x \in X$, then $H(x) = h(x) = (x, \sin(1/x))$, so that $\pi_2(H(x)) = \sin(1/x)$ as desired.

There is something more interesting about this last compactification. We constructed it by choosing an embedding $h: (0, 1) \to \mathbb{R}^2$ whose component functions were the functions $x$ and $\sin(1/x)$. Then we found that both the functions $x$ and $\sin(1/x)$ could be extended to the compactification. This suggests that if we have a whole collection of bounded continuous functions defined on $(0, 1)$, we might use them as component functions of an embedding of $(0, 1)$ into $\mathbb{R}^J$ for some $J$, and thereby obtained a compactification for which every function in the collection is extendable.

This idea is the basic idea behind the Stone-Čech compactification.

### 15.1.4 Uniqueness of Extensions into Hausdorff Spaces of Continuous Functions $f$ to $\text{dom } f$.

**Lemma 21.** Let $A \subseteq X$ and $f: A \to Z$ be a continuous map into a Hausdorff space $Z$. There is exists at most one extension of $f$ to a continuous function on $\overline{A}$.

**Proof.** Suppose for the sake of a contradiction that $g, g': \overline{A} \to Z$ are two different extension of $f$; choose $x$ so that $g'(x) \neq g(x)$. By Hausdorffness of $Z$, there exists disjoint nhbds $U$ and $U'$ of $g(x)$ and $g'(x)$ respectively. Choose a nhbd $V$ of $x$ so that $g[V] \subseteq U$ and $g'[V] \subseteq U'$. Now $V$ intersects $A$ in some point $y$ as $V$ is a nhbd of a point in the closure of $A$ in $X$; then $g(y) \in U$ and $g'(y) \in U'$ but also $g(y) = f(y) = g'(y)$ since $y \in A$ contradicting the fact that $U \cap U' = \emptyset$. ■

### 15.2 The Stone-Čech Compactification And Its Properties

#### 15.2.1 Tychonoff Spaces Have a Compactification To Which Every Continuous Function $g: X \to \mathbb{R}$ Extends.

**Theorem 15.72.** Let $X$ be a Tychonoff space. There exists a compactification $Y$ of $X$ having the property that every bounded continuous map $f: X \to \mathbb{R}$ extends uniquely to a continuous map $H: Y \to \mathbb{R}$.

![Diagram](https://via.placeholder.com/150)

We call this compactification of $X$ the **Stone-Čech compactification** of $X$. 

Proof. Let \((f_{\alpha})_{\alpha \in J}\) be the collection of all bounded continuous real-valued functions on \(X\), indexed by some index set \(J\). For each \(\alpha \in J\), choose a closed \(I_{\alpha}\) in \(R\) containing \(f_{\alpha}[X]\) (for example, \(I_{\alpha} = [\inf f[X], \sup f[X]]\)). Then define \(h: X \to \prod_{\alpha \in J} I_{\alpha}\) by the rule

\[ h(x) = (f_{\alpha}(x))_{\alpha \in J}. \]

By the Heine-Borel and Tychonoff’s theorem, \(\prod I_{\alpha}\) is compact. Because \(X\) is Tychonoff, the collection \((f_{\alpha})\) separates points from closed sets in \(X\) so that by the embedding theorem (Theorem 1.70), \(h\) is an embedding.

WLOG, we may suppose that \(X \subseteq Y\), where \(Y\) is the compactification of \(X\) induced by the embedding \(h\). Then by the lemma there is an embedding \(H: Y \to \prod I_{\alpha}\) that equals \(h\) when restricted to the subspace \(X\) of \(Y\) since \(Cl_{Y}(X) = Y\).

Given a bounded continuous real-valued function \(f\) on \(X\), we show it extends uniquely to \(Y\). The function \(f\) belongs to \((f_{\alpha})_{\alpha \in J}\) and so equals \(f_{\beta}\) for some \(\beta \in J\). Then \(\pi_{\beta} \circ H: Y \to I_{\beta}\) is the desired extension of \(f\) where \(\pi_{\beta}\) is the obvious projection map. For if \(x \in X\), we have

\[ \pi_{\beta}(H(x)) = \pi_{\beta}(h(x)) = \pi_{\beta}((f_{\alpha}(x))_{\alpha \in J}) = f_{\beta}(x). \]

Uniqueness of the extension follows from the preceding lemma. ■

15.2.2 Existence & Uniqueness of Extensions of Maps from a Tychonoff Space \(X\) into a Compact Hausdorff Space to the Stone-Čech Compactification of \(X\).

Theorem 15.73. Let \(X\) be a Tychonoff space and \(Y\) its Stone-Čech compactification. Then any continuous map \(f: X \to K\) of \(X\) into a compact Hausdorff space \(K\) extends uniquely to a continuous map \(g: Y \to K\).

\[
\begin{array}{ccc}
X & \xrightarrow{f} & K \\
\downarrow{\varphi} & & \downarrow{g} \\
Y & \xrightarrow{h} & K
\end{array}
\]

Proof. First, some reductions. As usual, WLOG we may suppose \(X \subseteq Y\), its Stone-Čech compactification. Note that \(K\) is Tychonoff as any compact space is normal and a \(T_{1}\) normal space is Tychonoff. Hence, \(K\) can be embedded in \([0,1]^{J}\) for some \(J\) by the embedding theorem. So WLOG we may as well assume that \(K \subseteq [0,1]^{J}\).

Having supposed \(K \subseteq [0,1]^{J}\), each component function \(f_{\alpha} = \pi_{\alpha} \circ f: X \to R\) of the map \(f\) is a bounded continuous real-valued function on \(X\); by Theorem 1.76, \(f_{\alpha}\) can be extended uniquely to a continuous map \(g_{\alpha}: Y \to R\) for each \(\alpha \in J\). Define \(g: Y \to R^{J}\) by \(g(y) = (g_{\alpha}(y))_{\alpha \in J}\); then \(g\) is continuous because \(R^{J}\) has the product topology. Now, in fact, \(g\) maps \(Y\) into the subspace \(K\) of \(R^{J}\). For continuity of \(g\) implies that

\[ g[Y] = \overline{g[X]} \subseteq \overline{g[X]} \subseteq K = K. \]

Thus, \(g\) is the desired extension of \(f\). \(g: Y \to K\) is unique because any other such extension \(g': Y \to K\) must satisfy \(\pi_{\alpha} \circ g' = \pi_{\alpha} \circ g\) for all \(\alpha \in J\), and the components uniquely determines the function, implying that \(g = g'\). ■

15.2.3 Any Two Stone-Čech Compactifications are Homeomorphic.

Theorem 15.74. Let \(X\) be a Tychonoff space. If \(Y_{1}\) and \(Y_{2}\) are two Stone-Čech Compactifications of \(X\), then there exists a unique homeomorphism \(h: Y_{1} \cong Y_{2}\) making the following diagram commute:

\[
\begin{array}{ccc}
X & \xrightarrow{\varphi} & Y_{1} \\
\downarrow{\psi} & & \downarrow{h} \\
Y_{2} & \xrightarrow{\psi} & Y_{2}
\end{array}
\]

Proof. Since \(Y_{1}\) and \(Y_{2}\) are compact Hausdorff, and \(\varphi: X \to Y_{1}\) and \(\psi: X \to Y_{2}\) continuous, there is a unique continuous function \(\Psi: Y_{1} \to Y_{2}\) such that \(\Psi \circ \varphi = \psi\) and similarly there is a \(\varphi: X \to Y_{1}\) there is a unique continuous function
15.3 Exercises

Φ: Y_2 \to Y_1 such that Φ \circ ψ = ϕ; this follows from Theorem 1.77. We contend that Φ and Ψ are inverse, proving the existence of the homeomorphism we desire. The composite Φ \circ Ψ: Y_1 \to Y_1 has the property that for each x ∈ X, Φ(Ψ(ϕ(x))) = ϕ(x). Hence, it is a continuous extension of the identity map on X. Since id_{Y_1}: Y_1 \to Y_1 is a continuous extension of id_ϕ[X], by Lemma 22, id_{Y_1} is the unique extension of the identity map on ϕ[X], forcing Φ \circ Ψ = id_{Y_1}. Similarly, Ψ \circ Φ: Y_2 \to Y_2 equals the identity map on Y_2. Thus, Φ = Ψ^{-1} so that Φ and Ψ are homeomorphisms. Set h = Ψ. Then h is unique by Theorem 1.77. ■

We are finally ready to give a formal definition of the Stone-Čech compactification of Tychonoff spaces. While we can define the Stone-Čech compactification of other types of spaces, Tychonoff spaces are the spaces which embed in its Stone-Čech compactification.

Definition (Stone-Čech Compactification of Tychonoff Spaces). The Stone-Čech compactification of a Tychonoff space X is a tuple comprised of a compact Hausdorff space βX along with an embedding ϕ: X → βX that has the following universal property: for any continuous map f: X → K where K is a compact Hausdorff space extends uniquely to a continuous map βf: βX → K.

Any other such space with the same property is homeomorphic to βX.

Remark. If a space is not Tychonoff, it still has a Stone-Čech compactification, but not in the usual sense. The only downside is that the map X → βX will not, in general, be an embedding! For instance, [0, 1) → R^2 by (cos(2πt), sin(2πt)) is not an embedding since its inverse is certainly not continuous! On the other hand, the same map restricted to (0, 1) is an embedding onto its image, clearly. This general version of the Stone-Čech compactification satisfies the same universal property.

15.3 Exercises

Exercise. The bounded continuous function g: (0, 1) → R, g(x) = cos(1/x) cannot be extended to the topologist’s sine curve. Define an embedding h: (0, 1) → [0, 1]^3 such that the functions x, sin(1/x) and cos(1/x) are all extendable to the compactification induced by h.

15.3.1 A Sufficient Condition For a Metrizable Space to Have a Metrizable Compactification.

Exercise. When does a metrizable space have a metrizable compactification?

[Hint: Use Urysohn’s metrization theorem.]

15.3.1.1 The Stone-Čech Compactification is “Maximal”.

Exercise. For any compactification Y of X, there exists a surjective closed map g: βX → Y from the Stone-Čech compactification of X to Y inducing the identity on X.

15.3.2 A Completely Regular Space is Connected iff βX is Connected.

Exercise. A completely regular space X is connected iff βX is connected.

[Hint: If A, B is a separation of X, let f(x) = 0 for x ∈ A and f(x) = 1 for x ∈ B.]
15.3.2.1 The Stone-Čech Compactification as a Functor.

The Stone-Čech compactification assigns to each completely regular space $X$ a compactification $\beta X$. Now let us assign to each continuous map $f : X \to Y$ of completely regular spaces the unique continuous map $\beta f : \beta X \to \beta Y$ that extends the map $\varphi \circ f : X \to \beta Y$ where $\varphi : Y \to \beta Y$ is the canonical inclusion.

Verify that if $1_X : X \to X$ is the identity map on $X$, then $\beta 1_X$ is the identity map on $\beta X$ and that if $f : X \xrightarrow{cont} Y, g : Y \xrightarrow{cont} Z$ between regular spaces, then $\beta (g \circ f) = \beta g \circ \beta f$. This makes $\beta$ into a functor from the category of completely regular spaces and continuous maps of such spaces to the category of compact Hausdorff spaces and continuous maps of such spaces.

15.3.3 Other Exercises

**Theorem 15.75.** Suppose $X$ is a Tychonoff space, $Y$ is a compact Hausdorff space, and $\phi \in C(X,Y)$. Then $\phi$ has a unique continuous extension $\tilde{\phi} \in C(\beta X,Y)$ to $\beta X$ such that $\tilde{\phi} \circ e = \phi$ where $e : X \to \beta X$ is the canonical inclusion. If $(Y,\phi)$ is a compactification of $X$, then $\phi$ is surjective; if also every $f \in BC(X)$ extends continuous to $Y$ (i.e., $f = g \circ \phi$ for some $g \in C(Y,Y)$), then $\phi$ is a homeomorphism.

**Exercise.** Suppose $X$ is Tychonoff. The set $M$ of nonzero algebra homomorphisms from $BC(X,R)$ to $R$, equipped with the topology of pointwise convergence, is homeomorphic to $\beta X$. (See Exercise 71 from Folland.)

**Exercise.** If $X$ is a second-countable normal space, then there exists a countable family $F \subseteq C(X,I)$ that separates points and closed sets. [Hint: Let $B$ be a countable base for the topology. Consider the set of pairs $(U,V) \in B \times B$ such that $U \subseteq V$, and use Urysohn’s lemma.]

**Embeddings in Manifolds**

**Definition.** A $n$-manifold is a second-countable, Hausdorff space that is locally Euclidean.

This section is more appropriately covered in other books on manifolds. The result is that any compact manifold can be embedded into $R^N$ for some $N \in N$.

**Exercises**

**The Point-Finite Shrinking Lemma.**

**Definition (Point-Finite Indexed Family).** An indexed family $\{A_\alpha\}$ of subsets of $X$ is said to be a point-finite indexed family if each $x \in X$ belongs to $A_\alpha$ for only finitely many values of $\alpha$.

**Lemma 22 (The Point-Finite Shrinking Lemma).** Let $X$ be a normal space; let $\{U_1,U_2,\ldots\}$ be a point-finite indexed open covering of $X$. Then there exists an indexed open covering $\{V_1,V_2,\ldots\}$ of $X$ such that $\overline{V_n} \subseteq U_n$ for each $n \in N$.

**Hausdorff Condition Necessary on a Manifold**

**Exercise.** The Hausdorff condition is an essential part of the definition of a manifold; it is not implied by the other parts of the definition. Consider the following space: Let $X = R \setminus \{0\} \cup \{p,q\}$. Topologize $X$ by taking as a basis the collection of all open intervals in $R$ that do not contain 0, along with all sets of the form $(-a,0) \cup \{p\} \cup (0,a)$ and all sets of the form $(-a,0) \cup \{q\} \cup (0,a)$ for $a > 0$. The space $X$ is called the **line with two origins**.

(a) Check that this is a basis for a topology.
(b) Show that each of the spaces $X \setminus \{p\}$ and $X \setminus \{q\}$ are homeomorphic to $R$.
(c) Show that $X$ is $T_1$ but not Hausdorff.
(d) Show that $X$ is second countable and locally Euclidean.
(e) Conclude that $X$ satisfies all conditions for being a 1-manifold except the Hausdorff condition.
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