Math 512 (Summer 2005): Homework #1

Instructor: Sergei Chmutov

Due Friday, June 24, before class

1. A 2π-periodic function is specified on the interval $[-\pi, \pi]$.

$$f(x) = \begin{cases} 0, & \text{if } -\pi \leqslant x < 0; \\ 1, & \text{if } 0 \leqslant x < \pi/2; \\ 0, & \text{if } \pi/2 \leqslant x \leqslant \pi; \end{cases}$$

Find the Fourier series of $f(x)$.

2. A 2π-periodic function $f(x) = x$ is specified on the interval $[0, 2\pi]$. Find the Fourier series of $f(x)$.

3. A 2π-periodic function $f(x) = x^3$ is specified on the interval $[-\pi, \pi]$. Find the Fourier series of $f(x)$.

4. A 2π-periodic function $f(x) = x^2/2$ is specified on the interval $[-\pi, \pi]$. Find the Fourier series of $f(x)$.

5. Use the Fourier series from previous problem to find the following sum

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots = ??? \ .$$