Math 512 (Summer 2005): Homework #3
Instructor: Sergei Chmutov
Due Friday, July 1, before class

Sec.2.7, p.75 #'s: **2, 6**; Sec.3.4, p.133 #'s: **2, 4**; **AND** the following problem.

Consider a semi-infinite string with fixed left end satisfying the differential equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} , \qquad x \geqslant 0, t \geqslant 0 ,$$

and the following boundary and initial conditions

$$u(0,t) = 0, \qquad u(x,0) = f(x) = \begin{cases} \dfrac{-x^2 + 10x - 24}{4}, & \text{if } 4 \leqslant x \leqslant 6 \\ 0, & \text{otherwise} \end{cases} , \qquad u_t(x,0) = 0 .$$

(**a**) Sketch the graph of the solution $u(x,t)$ for the following values of time t:

$$t = 0, \quad 1, \quad 2, \quad \frac{11}{4}, \quad 4 .$$

(**b**) Determine the times when the wave reaches the points $x = 2$, $x = 10$; the reflected wave reaches the point $x = 10$.

(**c**) The value $u(0.5, 2.5)$ is determined by two values $f(x_1)$ and $f(x_2)$ of the initial deflection $f(x)$ at certain points x_1 and x_2. Find x_1 and x_2. If $f(x_1) = a$ and $f(x_2) = b$, express $u(0.5, 2.5)$ in terms of a and b.