Math 512: Midterm #1

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NAME:

1. **(a)** Find the Fourier cosine series for the function $f(x) = -x + \pi/2$ on the interval $0 < x < \pi$.

(b) Using this Fourier series find the sum

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \dots$$

2. Find the general and steady-state solutions of the differential equation

$$y'' + 4y' + 13y = \cos t \ .$$

3. Consider a string of length 4 with fixed ends satisfying the differential equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}\,, \qquad 0 \leqslant x \leqslant 4, \quad t \geqslant 0\,,$$

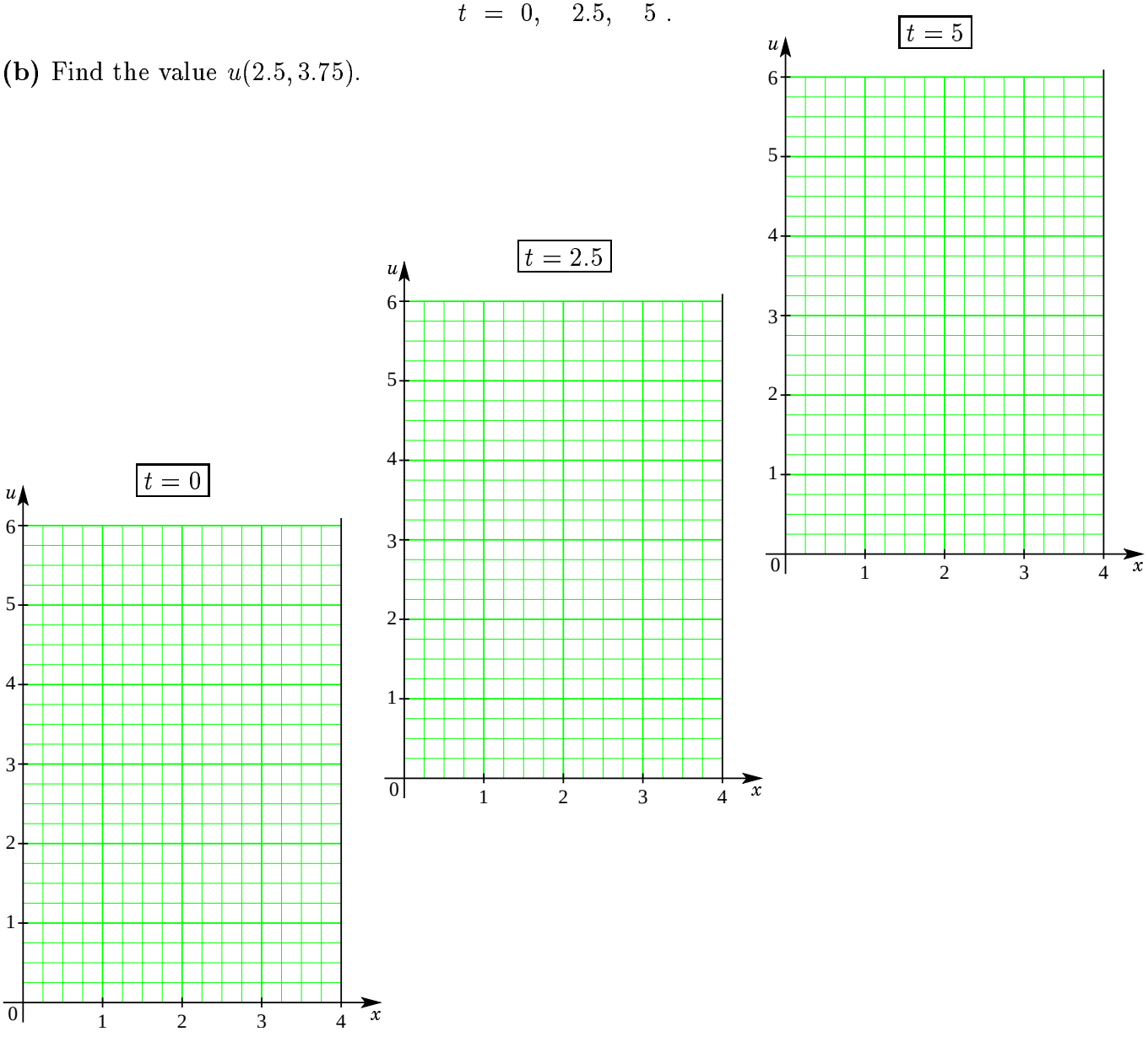
and the following boundary and initial conditions

$$u(0,t) = 0, \quad u(4,t) = 0, \qquad u(x,0) = f(x) = \begin{cases} 8x - 8, & \text{if} \quad 1 \leqslant x \leqslant 1.25 \\ \dfrac{-8x + 16}{3}, & \text{if} \quad 1.25 \leqslant x \leqslant 2 \\ 0, & \text{otherwise} \end{cases}, \quad u_t(x,0) = 0\,.$$

(a) Sketch the graph of the solution $u(x,t)$ for the following values of time t (use the grids below):

$$t \;=\; 0, \quad 2.5, \quad 5\,.$$

(b) Find the value $u(2.5, 3.75)$.



4. For a string of length π solve the following boundary value problem

$$
\begin{aligned}
&u_{tt} = u_{xx}, \qquad\qquad 0 \leqslant x \leqslant \pi, \quad t \geqslant 0 \\
&u(0,t) = 0, \quad u(\pi,t) = 0, \\
&u(x,0) = 0, \quad u_t(x,0) = \sin x \cos x \ .
\end{aligned}
$$

5. Solve the boundary value problem heat problem

$$u_t = 4u_{xx}, \qquad 0 \leqslant x \leqslant 1, \quad t \geqslant 0$$

$$u(0,t) = 0, \quad u(1,t) = 0,$$

$$u(x,0) = 2\sin(3\pi x)\cos(\pi x)\ .$$