Gauss diagram formulas for \textit{HOMFLYPT}

Gauss diagrams

A Gauss diagram is a collection of (based) oriented circles with
- ordered pairs (arrows) of distinct points
- sign of each pair.

Polyak - Viro (Gauss diagram) formulas \((A, b) = \frac{1}{2} \text{Tr} \text{Sw}

\[
<A, G> := \text{# of } A \text{-subdiagrams of } G, \quad <A, b> := (A, 2b)
\]

\[
<\begin{array}{c}
\bullet \\
\end{array}, G> = \text{# of } t \text{-arrows of } G \text{ from the second circle to the first one.}
\]

\[
<\begin{array}{c}
\bullet \\
\end{array}, G> = \text{# of } d \text{-arrows}
\]

\[
<\begin{array}{c}
\bullet \\
\end{array}, G> = \text{lk}(G)
\]

Shorter notation:

\[
<\begin{array}{c}
\bullet \\
\end{array}, G> = \text{coeff of } x^2 \text{ in } \text{polynomial}
\]

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\]

Gauss's theorem

Any Vassiliev knot invariant can be expressed by a Polyak-Viro formula.
Homflypt Polynomial

Skein relation

\[ a P(\uparrow) - a^{-1} P(\downarrow) = z P(\uparrow\downarrow) , \quad P(O) = 1 \]

\[ P(O-E) = (a^{-1} - \frac{a}{z})^{m-1} \]

Goussarov's Lemma

\[ P(L) |_{a = e^h} \quad \text{Taylor expansion in } h \text{ and } z \]

\[ \sum_{k,l} p_{k,l} h^k z^l \]

\( p_{k,l}(L) \) is a Vassiliev invariant of order \( \leq k + l \)

Goal: \( p_{k,l}(L) = \langle A_{k,l}, G_A \rangle \)

\((h,z)\) state model

\( A = \) Gauss diagram

A state \( S = \) subset of arrows of \( A \)

\[ W(A) = \sum_S \prod_{a \in A} <a(A S)> \left( \frac{e^h - e^{-h}}{2} \right)^{C(S) - 1} \]

\( k \quad l \)

double every arrow of \( S \)

\( C(S) = \# \text{ components} \)

**Diagram:**

First passage

\[ \sum A_{k,l} = \sum_{A} w_{k,l}(A) \cdot A \]
COEFFICIENTS OF THE HOMFLYPT POLYNOMIAL

\[ A_{0,2} = \otimes \otimes \text{;} \quad A_{2,0} = 0 \; \]

\[ A_{0,3} = 0 \; \quad A_{3,0} = -4A_{1,2} \; \]
\[ A_{1,2} = -2\left( \begin{array}{c}
\otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes - \otimes \otimes + \otimes \otimes + \\
+ \otimes \otimes - \otimes \otimes \end{array} \right) \; \]

\[ A_{0,4} = \begin{array}{c}
\otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \\
+ \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \\
+ \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \\
+ \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \\
+ \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \otimes \otimes + \\
\end{array} \; \]

\[ A_{2,2} = 78 \text{ terms.} \]
$k=0$, Conway polynomial (with Khovanov-Rozansky)

\[ A_{\otimes e} \]

double every arrow of $A$ \( (s=A) \)

pick those $A$ such that $c(A)=1$

contribution of an arrow: $\langle d | A | A \rangle = e^z$

$\text{to } w_{1R}(A)$

orientation of arrows is such that first approaching is to the head of arrow

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