Gauss diagram formulas for HOMFLYPT

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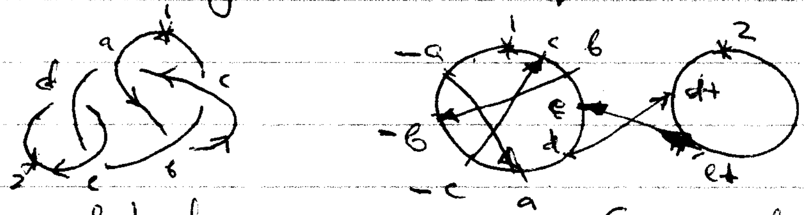
joint w/ M. Polyak.

Gauss diagrams

(Side board)

A Gauss diagram is a collection of (based) ordered, oriented circles with
— ordered pairs (arrows) of distinct points
— sign of each pair.



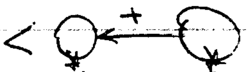
link diagram   Gauss diagram

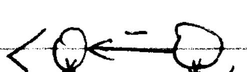
Polyak – Viro (Gauss diagram) formulas

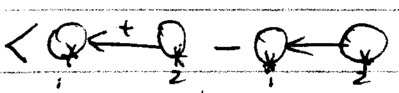
$(A, G) = \begin{cases} 1, & \text{if } A = G \\ 0 & \text{otherwise} \end{cases}$

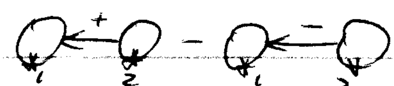
Pairing on the space spanned by all Gauss diagrams

$\langle A, G \rangle :=$ # of A ~~sub~~ subdiagrams of G. $\langle A, G \rangle := (A, \Sigma B)$

⟨ , G ⟩ = # of +-arrows of G from the second circle to the first one.

⟨ , G ⟩ = # of ~~of~~ −-arrows — " —

⟨ , G ⟩ = lk(G)

Shorter notation:  = 

⟨ , G ⟩ = ~~Coeff. of z¹ in~~ ~~∇~~(1, z) the Conway polynomial

Goussarov theorem

Any Vassiliev knot invariant can be expressed by a Polyak-Viro formula.

HOMFLYPT Polynomial

Skein relation

$$a\,P(\text{positive crossing}) - a^{-1}P(\text{negative crossing}) = z\,P(\text{smoothing}), \quad P(\bigcirc) = 1,$$

$$P(\underbrace{\bigcirc \cdots \bigcirc}_{m}) = \left(\frac{a-a^{-1}}{z}\right)^{m-1}$$

Goussarov's Lemma

$P(L)\big|_{a=e^h}$, Taylor expansion in h and z

$$= \sum_{k,\ell} p_{k,\ell}\, h^k z^\ell$$

$p_{k,\ell}(L)$ is a Vassiliev invariant of order $\leq k+\ell$

Goal: $p_{k,\ell}(L) = \langle \underbrace{A_{k,\ell}}_{???}, G_L \rangle$

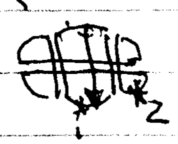
R. Side Board

(h,z)-state model

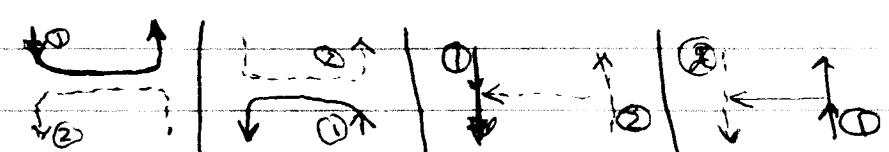
A = Gauss diagram

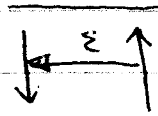
A state S = subset of arrows of A

$$W(A) = \sum_S \prod_{\alpha \in A} \langle \alpha | A | S \rangle \left(\frac{e^h - e^{-h}}{z}\right)^{c(S)-1} = \sum_{k,\ell} w_{k,\ell}(A)\, h^k z^\ell$$

double every arrow of S 

$c(S) :=$ # components

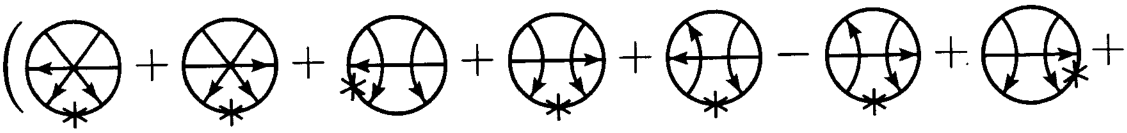


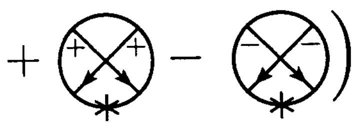
α \ First passage	(1)→(2)	(2)→(1)	(1)↓←(2)	(2)←(1)
 ε	$\varepsilon e^{-\varepsilon h} z$	0	$e^{-2\varepsilon h} - 1$	0

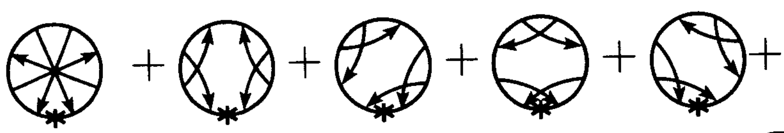
Th $A_{k\ell} = \sum_A w_{k,\ell}(A) \cdot A$

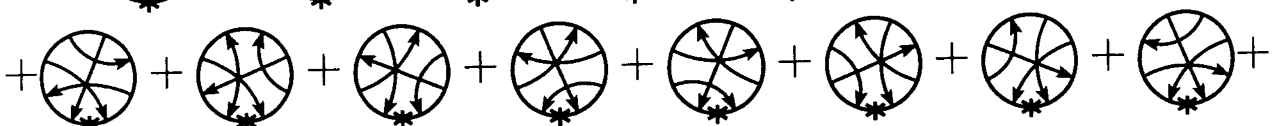
$A_{0,2} =$  ; $A_{2,0} = 0$;

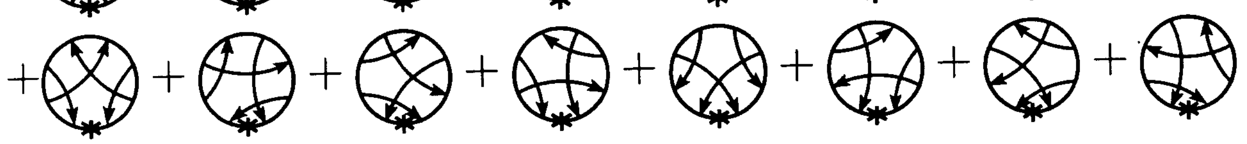
$A_{0,3} = 0$; $A_{3,0} = -4A_{1,2}$;

$A_{1,2} = -2$ 

 ;

$A_{0,4} =$ 



 ;

$A_{2,2} =$ 78 terms.

③ - slide

④

k = 0, <u>Conway polynomial</u> (with Khoury. Rossi)

$A_{0,e}$

double every arrow of A (S = A)

pick those A that $c(A) = 1$

contribution of an arrow: $\langle \alpha | A | A \rangle \in \mathbb{Z}$

to $W_{0,e}(A)$

orientation of arrows is such that first approaching is to the head of arrow

<u>Virtual links</u>