AMS #1053, Boca Raton, FL

Kamada-Miyazawa polynomial and ribbon graphs.

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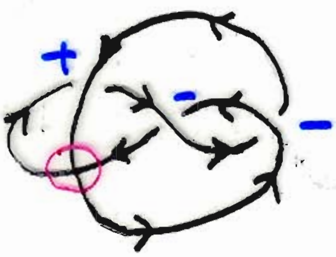
3:30 - 3:50 pm

Kamada - Miyazawa polynomial of virtual links

Y. Miyazawa (2004)
N. Kamada - Y.M. - Hiroshima Math. J. 35 (2005) 309-326

$$\Phi_D(A) + \Psi_D(A) \cdot h$$

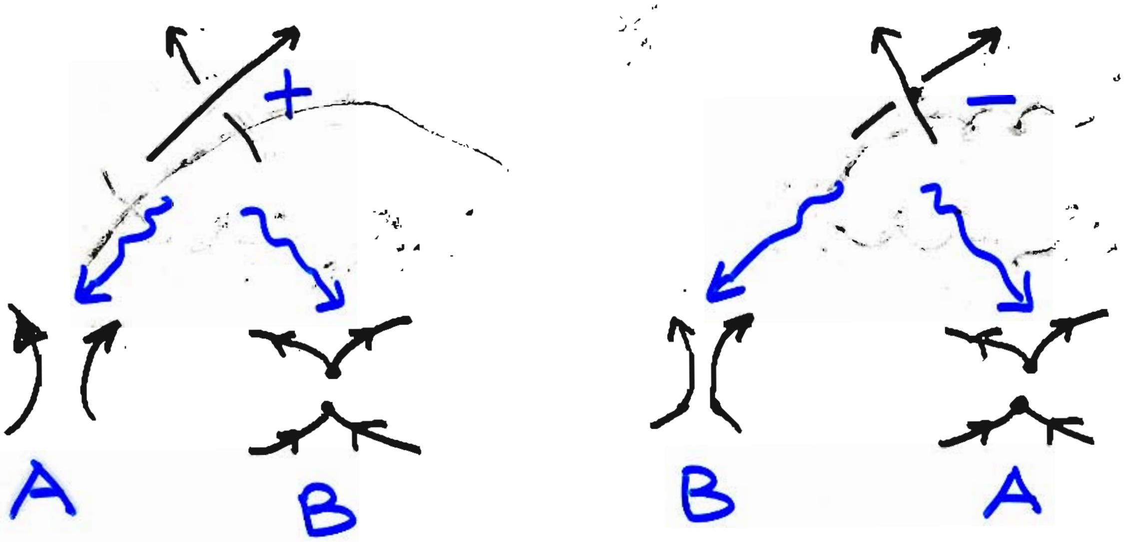
Idea: Split Kauffman's states in two types

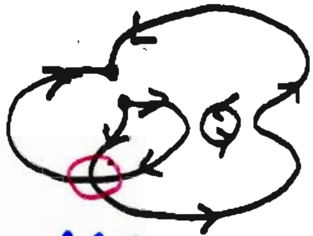


$w(K) = -1$

$$\Phi_D(A) + \Psi_D(A) = \langle D \rangle =$$

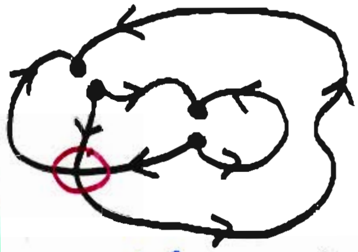
$$= \sum_S A^{\alpha(S)-\beta(S)}(-A^2-A^{-2})^{\delta(S)-1}$$





$i(S)=-1$

ψ $A^{-3}(-A^2-A^{-2})\underline{\underline{h}}$

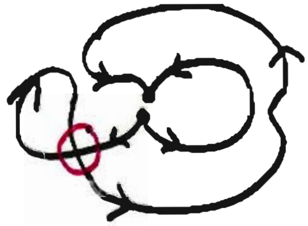


$i(S)=-1$

$A^{-1}\underline{\underline{h}}$ ψ

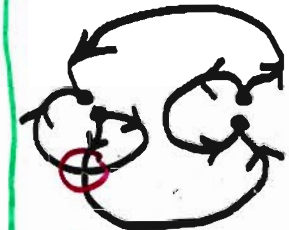
$i=+1$

$A^{-1}(-A^2-A^{-2})$



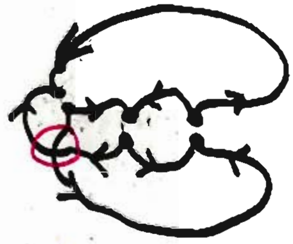
$i=+1$

A



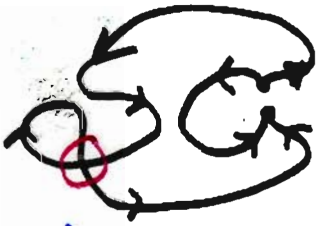
$i=-1$

$A^{-1}h$ ψ



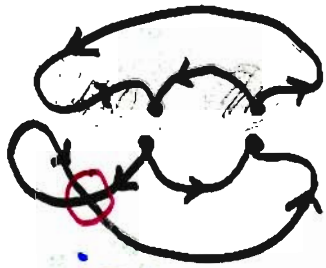
$i=+1$

A



$i=+1$

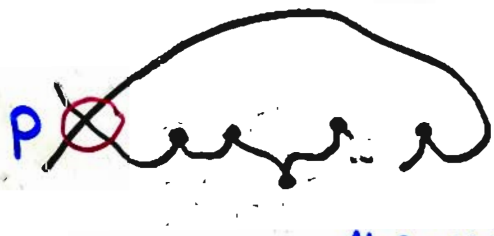
A



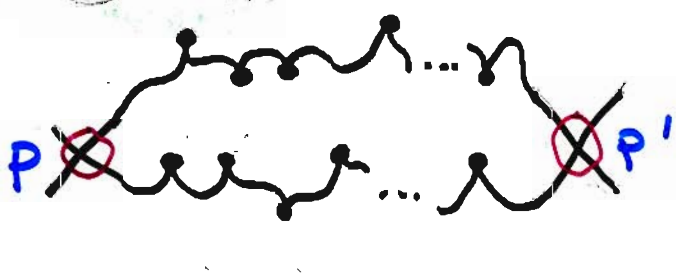
$i=+1$

$A^3(-A^2-A^{-2})$

$$i(S) = \prod_{p\in\{\otimes\}} i(p)$$

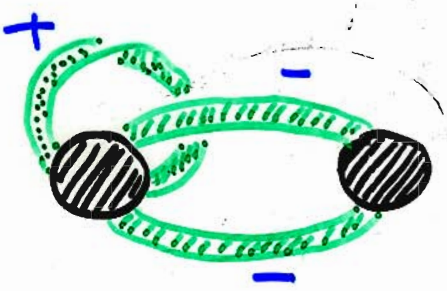


$i(p)=(-1)^{\# \text{cusps}}$



$i(p)\cdot i(p')=(-1)^{\# \text{cusps}}$

Ribbon graphs



Arrow presentation

$v(F) = \#$ vertices

$e(F) = \#$ edges

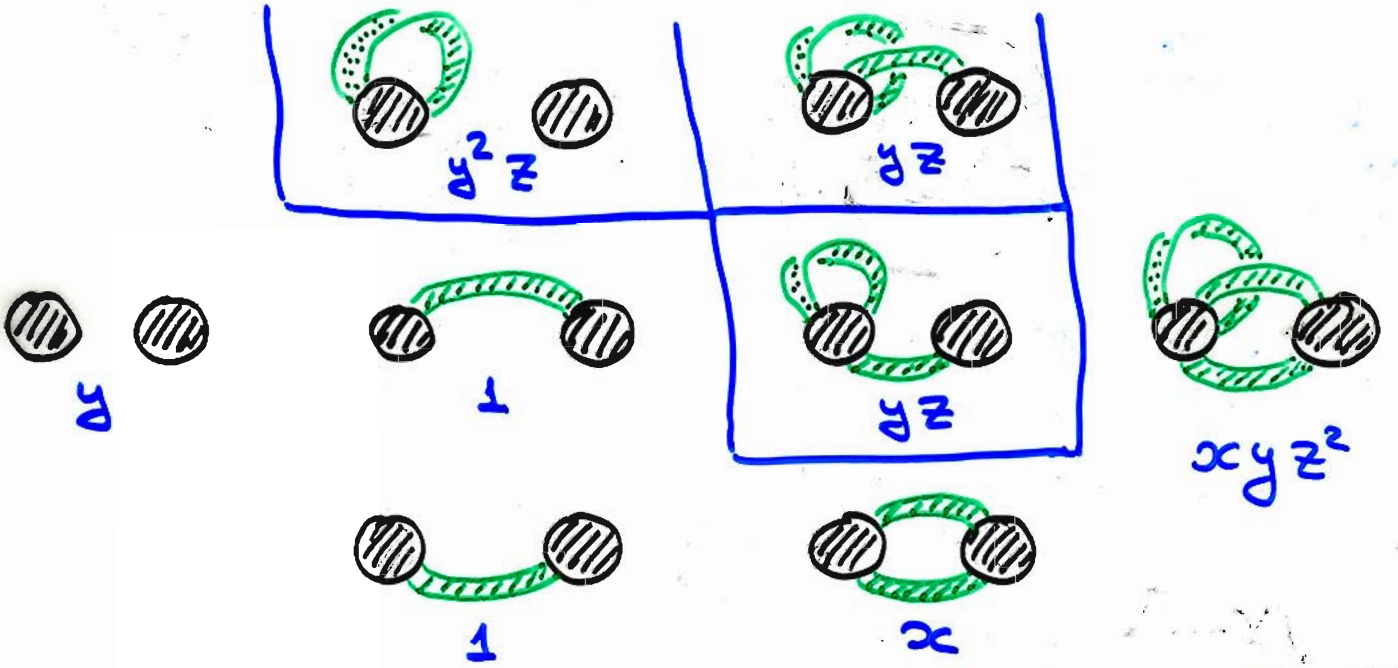
$k(F) = \#$ components

$r(F) = v(F) - k(F)$ rank

$n(F) = e(F) - r(F)$ nullity

$f(F) = \#$ of boundary components

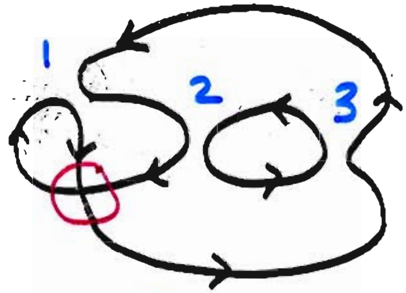
$s(F) = (e_-(F) - e_-(\bar{F}))/2$



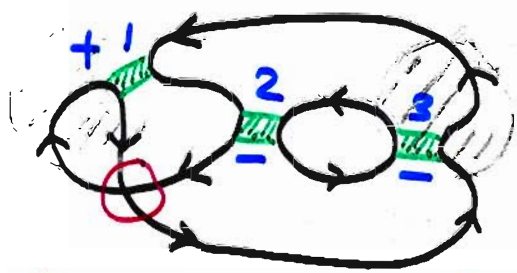
$$R_G(x,y,z) = \sum_F x^{r(G)-r(F)+s(F)} \cdot y^{n(F)-s(F)} \cdot z^{k(F)-f(F)+n(F)}$$

Bollobás–Riordan polynomial

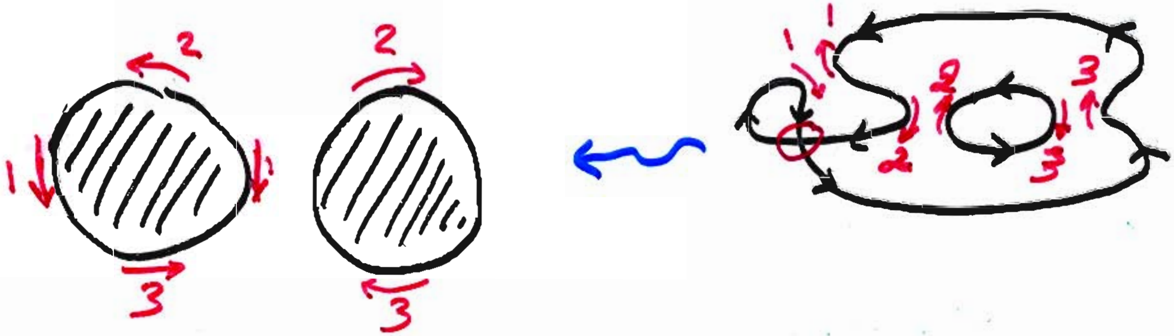
Ribbon graph G(s) associated with a state s



S



Arrow presentation



Virtual Thistlethwaite's Theorem

(S.Ch. J.Voltz JKFT 2008)

$$\langle D\rangle = A^{n-r} d^{k-1} R_{G(s)}(x,y,z)$$

$x = A^2 d$

$y = A^{-2} d$

$z = 1/d$

where

$$d = -A^2 - A^{-2}$$

Kamada-Miyazawa via Bollobás-Riordan

$$\Phi_D(A) = A^{n-r} d^{k-1} \cdot \underbrace{\frac{R(x,y,z)+R(x,y,-z)}{2}}_{\text{even part of } R} \Bigg|_{\substack{x=A^2 d\\ y=A^{-2}d\\ z=1/d}}$$

$$\Psi_D(A) = A^{n-r} d^{k-1} \cdot \underbrace{\frac{R(x,y,z)-R(x,y,-z)}{2}}_{\text{odd part of } R} \Bigg|_{\substack{x=A^2 d\\ y=A^{-2}d\\ z=1/d}}$$