Thistlethwaite's Theorem

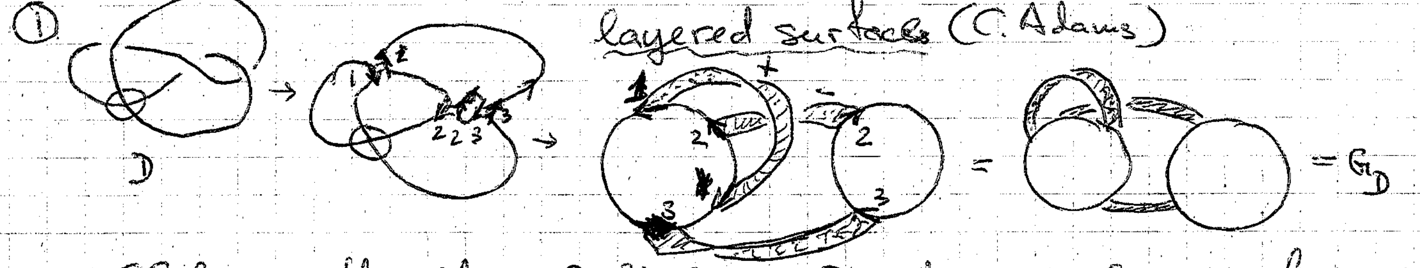
Wednesday
May 27, 2009
14:50–15:30
ICTP

$$V_L(t) \doteq \pm t^k\, T_\Gamma(t, t^{-1})$$

① J. Comb. Theory B, 99(3), 2009, 617–638

② Yuanan Diao, Gabor Hetyei
Relative Tutte polynomial for colored graphs and virtual knot Theory
http://www.math.uncc.edu/preprint/2008/2008_06.pdf

① layered surfaces (C. Adams)



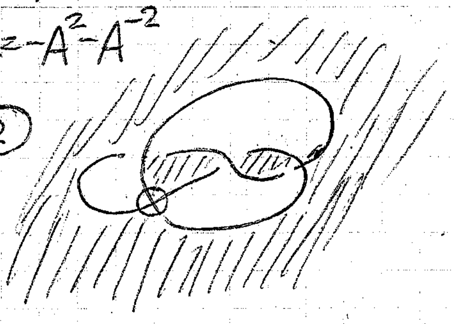
Slides with the Bollobás–Riordan polynomial and the example.

Th.

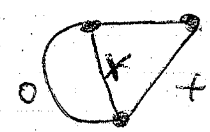
$$\langle L \rangle = A^{n(G)-r(G)}(-A^2-A^{-2})^{k-1} \cdot R_G\left(-A^4-1,\ -1-A^{-4},\ \frac{1}{-A^2-A^{-2}}\right)$$

$B := A^{-1}$
$d := -A^2-A^{-2}$

②



$e \notin H$



$$T_H(G) := \begin{cases} y_\pm T_H(G \setminus e) + x_\pm T_H(G/e) & \text{if } e \text{ is not a bridge nor a loop} \\ X_\pm T_H(G/e) & \text{if } e \text{ is a bridge} \\ Y_\pm T_H(G-e) & \text{if } e \text{ is a loop} \end{cases}$$

$T_H(H) = d^{\#\text{comp.} - 1}$

$$T_H(G) = y^2X + y^2x + yx^2 + xyX + x^2Y$$

$X_+ = -A^{-3}$	$X_- = -A^3$
$Y_+ = -A^3$	$Y_- = -A^{-3}$
$x_+ = A$	$x_- = A^{-1}$
$y_+ = A^{-1}$	$y_- = A$

$d = -A^2-A^{-2}$

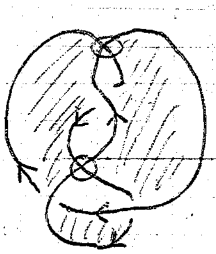
Th. $\langle L \rangle = T_H(G)$ subst.

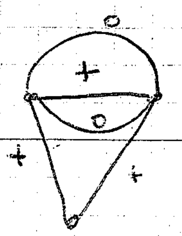
4T-relation:

$$X_+y_- - X_-y_+ = x_+y_- - x_-y_+ = x_+Y_- - x_-Y_+$$

$$\langle K \rangle = -A^{-5} + A^{-1} + A - A^{-3} - A^5$$

$y = y_+,\ x = x_+$



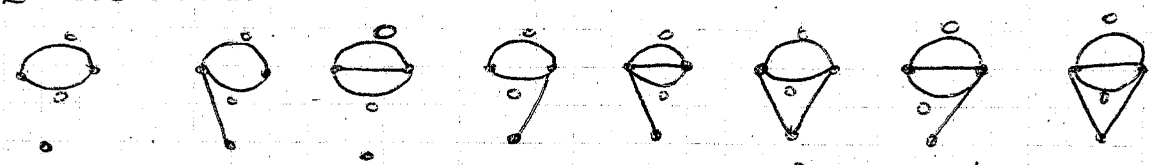


$y_+ (\cdot) + x_+ (\cdot) = y\left(y (\cdot) + x (\cdot)\right) + x\left(y (\cdot) + x (\cdot)\right)$

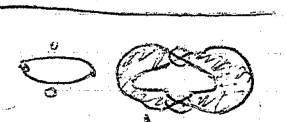
$= y^2 X (\cdot) + yx\left((\cdot) + x (\cdot)\right) + xyX (\cdot) + x^2 Y (\cdot)$?

$= (y^2X + y^3x)(\cdot) + (yx^2 + xyX + x^2Y)(\cdot)$?

8 состояний



y^2X $\quad y^3x \quad x^2Y$? $\quad yx^2 \quad xyX$

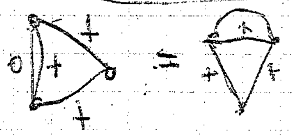


$\psi = d \quad |\Gamma|_0 = 2$

$|\Gamma|_0 = 1 \quad \psi = 1$

$\langle K \rangle = -A^{-3} + A^{-7} + A^{2}(-A^{3})$

$= -A^{-3} + A^{-7} - A^{5}$



$\sim (y^3X + y^2x)(\cdot) + (yx^2 + xyX + x^2Y)(\cdot)$

$\psi(\cdot) = 1 \qquad \psi(\cdot) = 1$

$= y^2X + y^2x + yx^2 + xyX + x^2Y =$

$\langle K \rangle = A^{-2}(-A^{-3}) + A^{-2}\cdot A + A^{-1} A^{2} + A\cdot A^{-1}(-A^{-3}) + A^{2}(-A^{3}) =$

$= -A^{-5} + A^{-1} + A - A^{-3} - A^{5}$

$J_K(t) = (-A^{-3})^3 \langle K \rangle$

$= A^{-12} - A^{-16} + A^{-4}\Big|_{A = t^{-1/4}}$

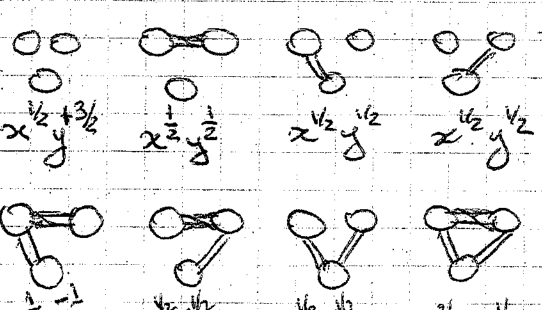
$= t^3 - t^4 + t$

Tutte ~~Done~~

$y^2X + y^2x + \cancel{yx^2} + xyX + x^2Y$

нет

8 состояний для G

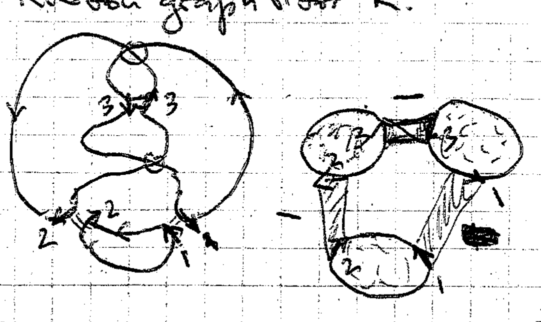


$x^{1/2}y^{3/2} \quad x^{1/2}y^{1/2} \quad x^{1/2}y^{1/2} \quad x^{1/2}y^{1/2}$

$x^{1/2}y^{-1/2} \quad x^{1/2}y^{-1/2} \quad x^{1/2}y^{-1/2} \quad x^{3/2}y^{-1/2}$

$\Rightarrow x^{1/2}y^{3/2} + 3x^{1/2}y^{1/2} + 3x^{1/2}y^{-1/2} + x^{3/2}y^{-1/2}$

Ribbon graph G for K.



$[K] = A^3 \left(\frac{B}{A}\right)^{3} \left(\frac{1}{A}\right)^{4} \left(\left(\frac{Ad}{B}\right)^{1/2} \left(\frac{Bd}{A}\right)^{3/2} + 3\left(\frac{Ad}{B}\right)^{1/2}\left(\frac{Bd}{A}\right)^{1/2} + 3\left(\frac{Ad}{B}\right)^{1/2}\left(\frac{Bd}{A}\right)^{-1/2} + \left(\frac{Ad}{B}\right)^{3/2}\left(\frac{Bd}{A}\right)^{-1/2} \right) =$

$= AB^{2}\left(A^{-1}Bd^{2} + 3d + 3AB^{-1} + A^{2}B^{-2}d\right)$

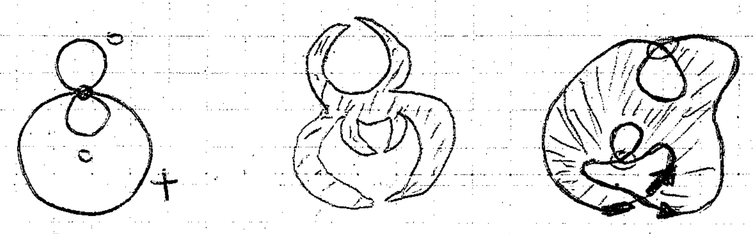
$B = A^{-1}$

$d = -A^{2} - A^{-2}$

$\langle K \rangle = A^{-1}\left(A^{-2}(-A^{2} - A^{-2})^2 + 3(-A^{2} - A^{-2}) + 3A^{2} + A^{4}(-A^{2} - A^{-2})\right)$

$= A^{-1}\left(A^{2} + \cancel{2A^{-2}} + A^{-6} - \cancel{3A^{2}} - \cancel{3A^{-2}} + \cancel{3A^{2}} + A^{4}\right) = A^{-1}\left(A^{2} - A^{-2} + A^{-6} - A^{6} - A^{2}\right)$?

$= -A^{-3} + A^{-7} - A^{5}$ OK Thistle-thwaite



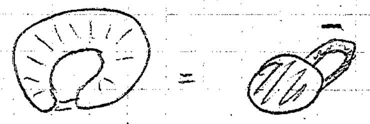
D-H: $Y 8^{\circ}_{\circ} = Y$

$\langle K \rangle = -A^3$

$J_k = (-A^{-3})\cdot(-A^3) = 1$

OK

Ch:



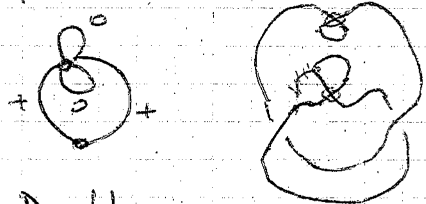
2 состояния

$x^{-1/2} y^{1/2}$ $\quad$ $x^{1/2} y^{1/2}$

$[K] = A\left(\cancel{\frac{Ad}{B}}\,\cancel{\frac{Bd}{A}}\,\frac{1}{\cancel{d^2}}\cdot\left(\left(\frac{Ad}{B}\right)^{-1/2}\left(\frac{Bd}{A}\right)^{1/2} + \left(\frac{Ad}{B}\right)^{1/2}\left(\frac{Bd}{A}\right)^{1/2}\right)\right) = A\left(A^{-1}B + d\right)$

$\langle K \rangle = A\left(\cancel{A^{-2}} - A^2 - \cancel{A^{-2}}\right) = -A^3$

Пример 2



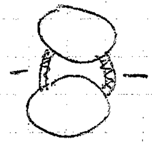
D-H: $yX8 + xY8$

$\langle K \rangle = A^{-1}\cdot(-A^{-3}) + A(-A^3)$

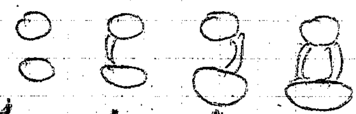
$= -A^{-4} - A^4$

OK

Ch:



4 состояние



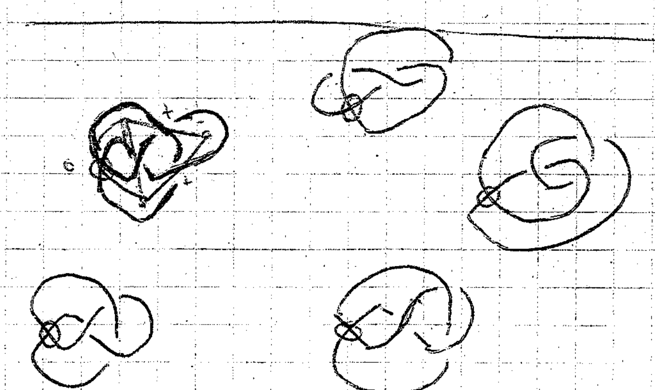
$y \quad 1 \quad 1 \quad x$

$[K] = \cancel{A^2}\,\cancel{\frac{Ad}{B}}\left(\cancel{\frac{Bd}{A}}\right)^{2}\frac{1}{\cancel{d^3}}\left(\frac{Bd}{A} + 2 + \frac{Ad}{B}\right)$

$= AB\left(\frac{Bd}{A} + 2 + \frac{Ad}{B}\right) = B^2 d + 2AB + A^2 d$

$\langle K \rangle = (A^2 + A^{-2})(-A^2 - A^{-2}) + 2 = -A^4 \cancel{-2} - A^{-4} \cancel{+2}$

$= -A^4 - A^{-4}$



$R = x + 2 + y + xyz^2 + 2yz + y^2 z$

$[K] = A^3 \cdot \frac{Ad}{B}\left(\frac{Bd}{A}\right)^2 \cdot \frac{1}{d^3}\left(\frac{Ad}{B} + 2 + \frac{Bd}{A} + 1 + \frac{2B}{A} + \frac{B^2 d}{A^2}\right)$

$= A^2 B\left(\frac{Ad}{B} + 3 + \frac{Bd}{A} + \frac{2B}{A} + \frac{B^2 d}{A^2}\right)$

$\langle K \rangle = A\left(A^2(-A^2 - A^{-2}) + 3 + A^{-2}(-A^2 - A^{-2}) + 2A^{-2} + A^{-4}(-A^2 - A^{-2})\right)$

$= A\left(-A^4 - 1 + 3 - 1 - A^{-4} + 2A^{-2} - \cancel{A^{-2}} - A^{-6}\right)$

$= -A^5 + A - A^{-3} + A^{-1} - A^{-5}$