<u>Combinatorics of virtual links</u>

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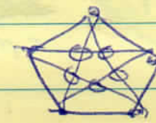
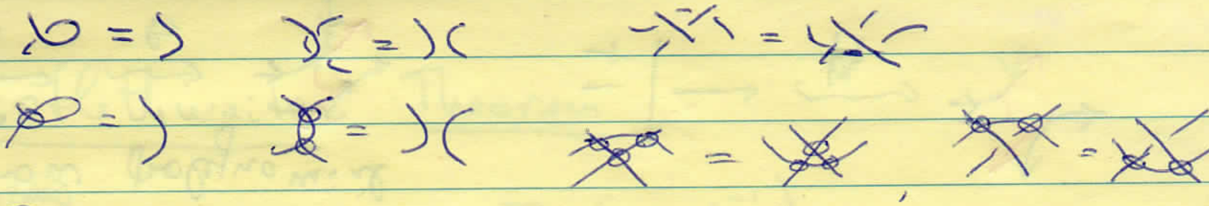
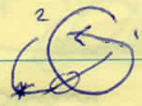
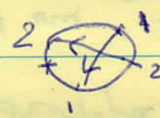
① <u>Virtual</u> <u>links</u> $K_5 =$ 

Reidemeister moves:

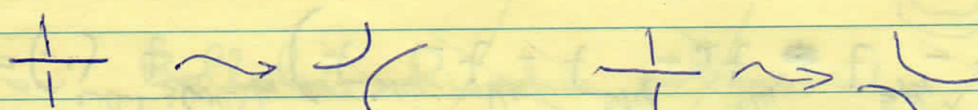




Gauss diagram



② <u>Jones</u> <u>polynomial</u>



A-splitting B-splitting

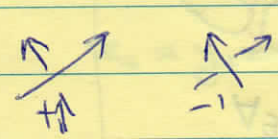
s is state (choice of A or B)

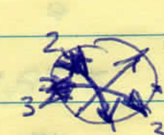
<u>Kauffman</u> ~~the~~ <u>bracket</u>

$$[L](A,B,d) := \sum_s A^{\alpha(s)} B^{\beta(s)} d^{\delta(s)-1}$$

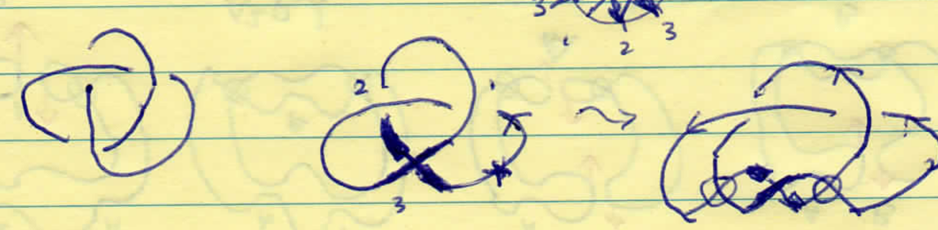
$\delta(s) = $ # of state circles.

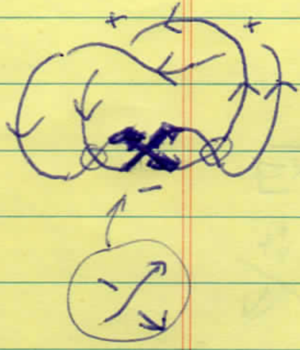
<u>Jones</u>: $J_L(t) := (-1)^{w(L)} t^{3w(L)/4} [L](A,B,d)\Big|_{A=t^{-1/4},\, B=t^{1/4},\, d=t^{1/2}-t^{-1/2}}$

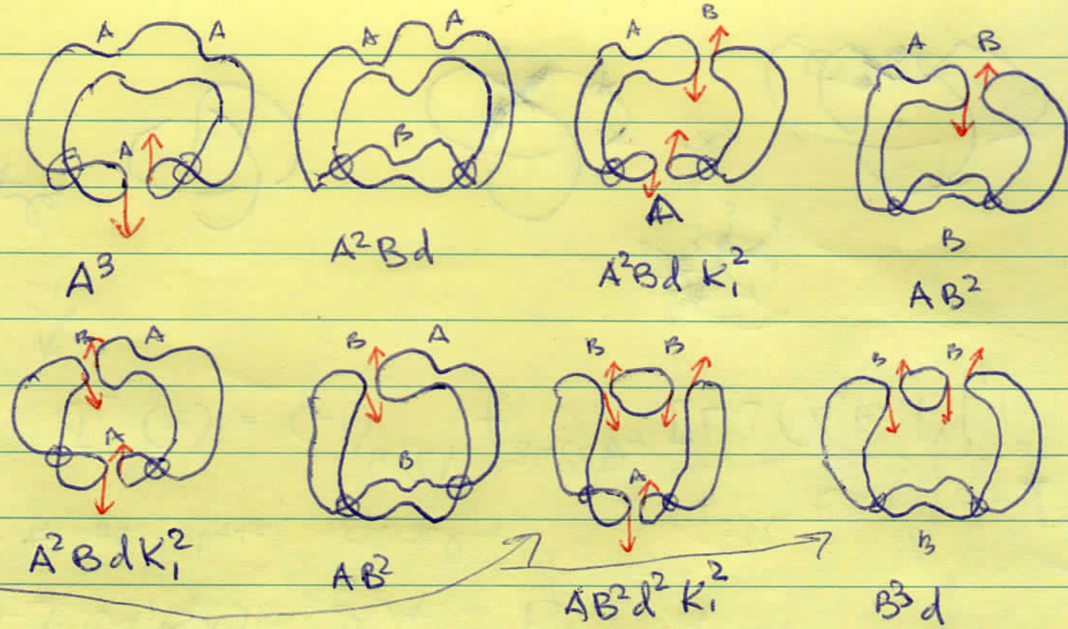




<u>Example</u>







$$J_L = (-1)^{w(L)} t^{3w(L)/4}\Big(t^{-3/4} + t^{-1/4}(-t^{1/2}-t^{-1/2}) + t^{-1/4}(-t^{1/2}-t^{-1/2})K_1^2$$

$w(L) = +1$

$$+ t^{1/4} + t^{-1/4}(-t^{1/2}-t^{-1/2})K_1^2 + t^{1/4} + t^{1/4}(-t^{1/2}-t^{-1/2})^2K_1^2 + t^{3/4}(-t^{1/2}-t^{-1/2})\Big)$$

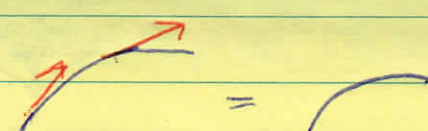
$$= (-1)^{w(L)} t^{3w(L)/4}\Big(\cancel{t^{-3/4}} - \cancel{t^{1/4}} - \cancel{t^{-3/4}} + \cancel{2t^{1/4}} - t^{5/4} - \cancel{t^{1/4}} \Big) +$$

$$+ \left(-\cancel{2t^{1/4}} - 2t^{-3/4} + t^{5/4} + \cancel{2t^{1/4}} + t^{-3/4} \right) K_1^2 \Big)$$

Show these one for example

③ Arrow polynomial

Cancellation of arrows



$$[L]_A(A,B,d) = \sum_S A^{\alpha(S)} B^{\beta(S)} d^{\delta(S)-1} \prod_{f\in S} K_{c(f)}$$

$c(f) = \frac{1}{2}\#$ arrows of $\overset{\text{a state}}{\text{circle}}$ f after cancellations

Example above

④ Thistlethwaite Theorem

$$J_L(t) \sim T_{G_L}(\ast t, \ast t^{-1})$$

ribbon graphs

arrow Bollobás-Riordan polynomial

$$\langle\langle G\rangle\rangle_A := \sum_{F\subseteq E(G)} \Big(\prod_{e\in F} x_e\Big)\Big(\prod_{e\notin F} y_e\Big) X^{r(G)-r(F)} \cdot Y^{n(F)} \cdot Z^{k(F)+n(F)-bc(F)} \cdot \prod_{f\in\partial(F)} K_{c(f)}$$

Arrow thistlethwaite theorem:

$$[L]_A(A,B,d) = A^{e}\Big(X^{k} Y^{v} Z^{v+1} \langle\langle G_L^{s}\rangle\rangle_A(X,Y,Z)\Big|_{\substack{X=\frac{Ad}{B}\\ Y=\frac{Bd}{A}\\ Z=1/d}}\Big)$$

$$x_+ = y_+ = 1, \quad x_- = \frac{A}{B}, \quad y_- = \frac{B}{A}$$