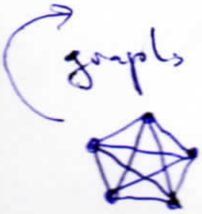
Graphs on surfaces via planar graphs

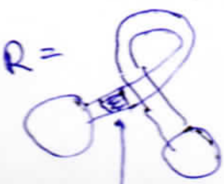
joint with C. Butler

Arnold-75
Monday, Nov. 17 2012
15:05–15:35

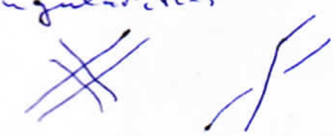
graphs on surfaces $\Longleftrightarrow$ ribbon graphs



R =

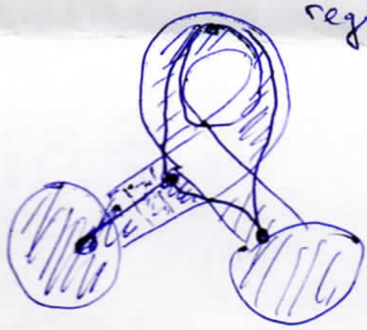


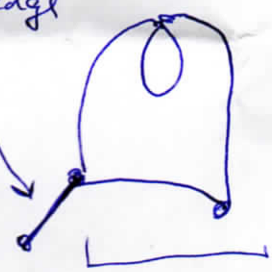
singularities



Medial graphs of ribbon graphs.

regular edge





= G

0-edges, $H \subset E(G)$

Theorem

$$X^{\alpha} Y^{\beta} T_{G,H}(X,Y) = B_R\left(X, Y, \frac{1}{\sqrt{XY}}\right)$$

Relative Tutte polynomial

Bollobás–Riordan polynomial.

$$\alpha = k(G) - k(R) - \beta, \quad \beta = -\frac{1}{2}\left(v(R) - v(G)\right)$$

k: # conn. comp.

v: # vertices

Bollobás–Riordan polynomial

$$B_R(X,Y,Z;\{x_e, y_e\}) = \sum_{F \subset E(R)} \Big(\prod_{e \in F} x_e \prod_{e \notin R \setminus F} y_e\Big) X^{k(F)-k(R)} Y^{n(F)} Z^{k(F)+n(F)-bc(F)}$$

$n(F) := e(F) - v(F) + k(F)$

$e(F)$: # edges of F

$bc(F) :=$ # conn components of ∂F

For virtual links L
$x_+ = y_+ = 1$, $x_- = A/B$, $y_- = B/A$

$$[L](A,B,d) = A^n B^{e-n} d^{k-1} B_R\left(\frac{Ad}{B}, \frac{Bd}{A}, \frac{1}{d}\right)$$

Relative Tutte polynomial (Y. Diao, G. Hetyei '08) (2

$$T_{G,H} := \sum_{F \subseteq G \setminus H} \Big(\prod_{e \in F} x_e \prod_{e \in \bar{F}} y_e \Big) X^{k(F \cup H) - k(G)} \, Y^{n(F)} \cdot \Psi(H_F)$$

$$H_F := (F \cup H)/F$$

$$\Psi(H_F) = \sqrt{XY}^{\,\delta(H_F) - k(H_F)} \cdot \sqrt{\frac{X}{Y}}^{\,v(H_F) - k(H_F)}$$

$\delta(H_F)$ = # circuits that immerse to the medial graph of H_F

Example

$F \cup H$

$\Longrightarrow H_F$

medial graph

$\delta(H_F) = 1$

Thistlethwaite's theorem