Université Montpellier — Thursday, May 26, 11 am

Jones polynomial for virtual knots

Dessins d'enfants A. Grothendieck '1984

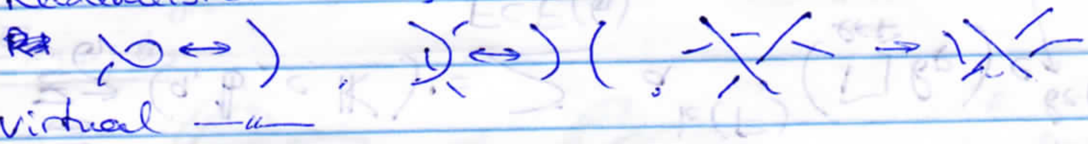
Thistlethwaite's theorem (1987)

$L =$  $=: \Gamma_L$

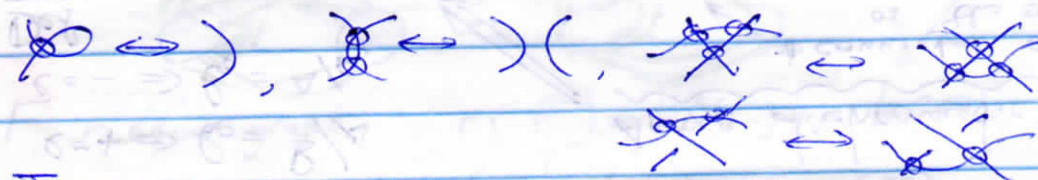
$$\boxed{J_L(t) = \pm t^N T_L(-t, -t^{-1})}$$

Virtual knot theory (Kauffman '1999)

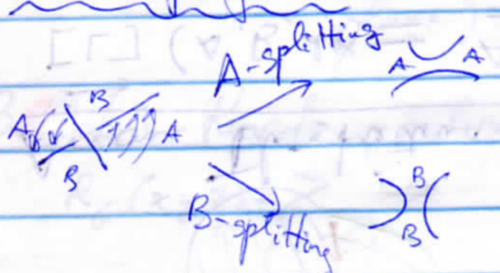
Reidemeister moves



virtual — " —



Jones polynomial:



State S = choice of either A- or B-splitting at each crossing.

$\alpha(S) := \#$ A-splittings

$\beta(S) := \#$ B-splittings

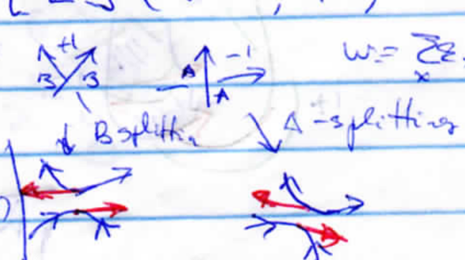
$\delta(S) := \#$ circles in $S = \#\partial S$

$$[L](A,B,d) := \sum_{S \text{ states}} A^{\alpha(S)} B^{\beta(S)} d^{\delta(S)-1}$$

$$J_L(t) := (-1)^{w(L)} t^{3w(L)/4} [L](t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2})$$

Arrow polynomial

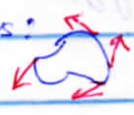
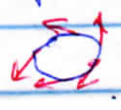
(H. Dye, L. Kauffman '2009)



$w = \sum_x \varepsilon_x$

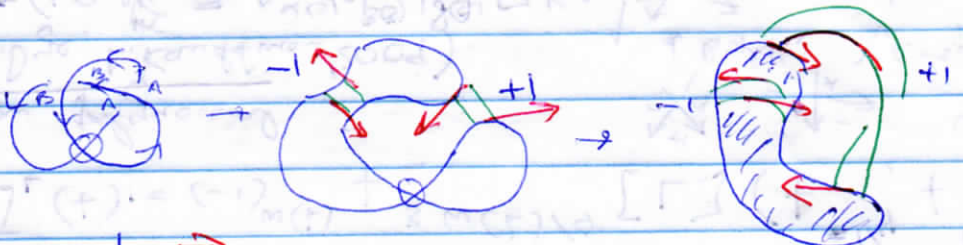
$$[\vec{L}]_A(A,B,d,K) := \sum_{\text{states } S} A^{\alpha(S)} B^{\beta(S)} d^{\delta(S)-1} \prod_{\ell \in \partial S} K_{i(\ell)}$$

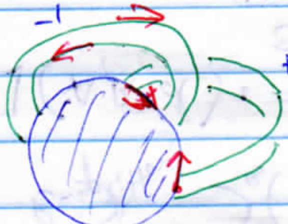
Cancellation of arrows:

 $i(\ell) = 1$  $i(\ell) = 1/2$  $i(\ell) = 2$

$i(\ell) = \frac{1}{2} \#$ arrows $\quad (K_0 := 1)$

Arrow signed ribbon graph

L 

$=$  $=: \vec{G}_L$

Arrow Thistlethwaite's theorem

$$\overrightarrow{[L]}(A,B,d,\mathbb{K}) = \frac{A^{e_+}B^{e_-}}{d}\, \vec{Z}_{\vec{G}_L}(1, \mathbb{b}, d, \mathbb{K})$$

Arrow dichromatic polynomial

$e=+ \Rightarrow b_e = B/A$

$e=- \Rightarrow b_e = A/B$

Def

$$\vec{Z}_{\vec{G}}(a, \mathbb{b}, c, \mathbb{K}) := \sum_{F\subseteq E(G)} a^{k(F)} \left(\prod_{e\in F} b_e\right) c^{bc(F)} \prod_{\ell \in \partial F} K_{i(\ell)}$$

$k(F)$: # connected component of the spanning subgr F

$bc(F)$: # boundary components of F

Example

F	$\emptyset$	$\{-1\}$	$\{+1\}$	$\vec{G}_L$
	acK_1	$a\cdot b_- c^2 K_1$	ab_+c	$ab_+b_-cK_1$

$\left(\frac{AB}{d}\right)$

S				
	A^2dK_1	ABK_1	ABK_1	B^2