For Professor Chmutov
-notes for my part of the talk - Avi S.

Bill Mance, James Sharpack, Avi Silterra 8/22/05

We now are close to having enough to present some relations between graph operations and the change in the tutte polynomial.

Tutte poly def (for either matroids or graphs)
poly in 2 indeterminates x, y

$$T(G,x,y) = \begin{cases} x\,T(G-e,x,y) & e \text{ a } \cancel{\text{bridge}}\ \text{Loop} \\ y\,T(G/e,x,y) & e \text{ bridge} \\ x\,T(G-e) + y\,T(G/e) & \text{o/w} \end{cases}$$

$M=(S,\rho)$ rank defn of matroid

or $$T(M,x,y) = \sum_{A \subseteq S} (x-1)^{\rho(S)-\rho(A)} (y-1)^{|A|-\rho(S)}$$

Interested in what occurs to ~~the~~ the tutte poly of graph under following -

① - "lengthen" each edge of graph by inserting one xtra vertex

② - lengthen by k extra vertices @ each edge

③ - "Double" each edge

④ - "k tuple" each edge

First ① - Idea! We'll express resulting graph $\otimes H$ for some fixed H. ie as a tensor product w/ another graph H
What graph H to choose?
Triangle! (3 cycle).
Note that point chosen for tensor prod irrelevant for triangle.

cont

Have following relations
(see Tangles & Matroids - Stephen Huggett)
for $\otimes$ of matroids, where $M=(S,p)$,
T_C, T_L, A, B certain poly's in x, y

$$T(M \otimes N) = T_C(N)^{|S|-p(S)}\, T_L^{\,p(S)}\, T(M, A, B)$$

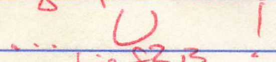
$$A = \frac{(x-1)T_C(N) + T_L(N)}{T_L(N)}$$

$$B = \frac{T_C(N) + (y-1)T_L(N)}{T_C(N)}$$

and

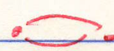
$$(x-1)T_C(N) + T_L = T(N-d)$$
$$T_C(N) + (y-1) = T(N/d)$$

If graph tensoring is H, then matroid N of H is



Note choice of 2,3 d is irrelevant for H (we are switching between Tutte poly on matroids & on graphs w/o explicit mention).

N-d is matroid of , which has tutte poly x^2

N/d is matroid of , has tutte poly $x+y$

so, we can take
$A = x^2$, $B = \frac{x+y}{x+1}$

$T_C = x+1$, $T_L = 1$ to satisfy equations
and obtain $T(M \otimes U_{2,3}) = (x+1)^{|S|-p(S)}\, T\left(M, x^2, \frac{x+y}{x+1}\right)$

Operation 2 is similar, for inserting k new vertices ~~At~~ tensor with a k+2 cycle.
Then take $A = x^{k+1}$, $B = \dfrac{y + \sum_{i=1}^{k} x^i}{\sum_{i=1}^{k} x^i}$, $T_C = \sum_{i=1}^{k} x^i$, $T_L = 1$

Note this corrects typo in paper. Similar formula for other operations