

M. CAP KHOURY

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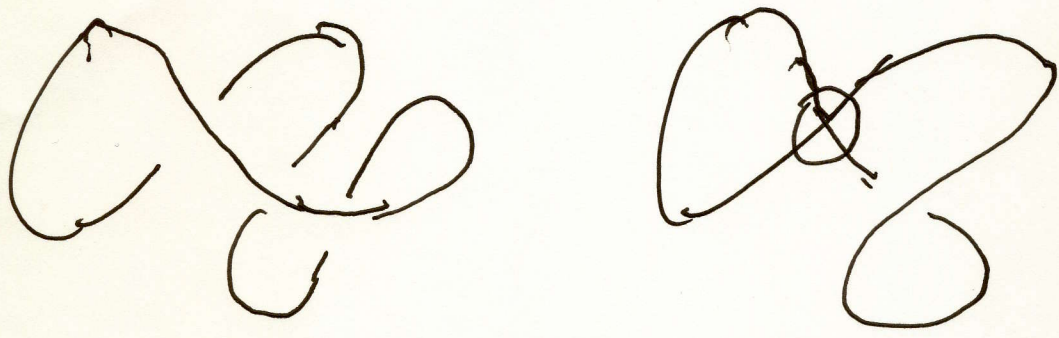
DO NOT TURN OFF THE ROOM LIGHTS

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POLYAK - VIRO FORMULAS FOR VASSILIEV INVARIANTS

1. VASSILIEV INVARIANT
 2. GAUSS DIAGRAM
 3. ARROW DIAGRAM FORMULA
-



$$\text{Link with crossing and arrows} = + \text{Link with crossing resolved} - \text{Link with crossing resolved}$$

$$\text{Crossing with arrows} = \text{Crossing with arrows resolved} - \text{Crossing with arrows resolved}$$

VASSILIEV INVARIANT OF ORDER n

IS A KNOT INVARIANT WHICH VANISHES ON ANY KNOT WITH $>n$ SINGULAR CROSSING

INVARIANTS OF ORDER 0 = constant function

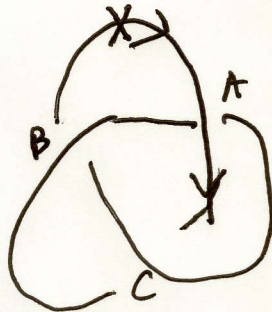
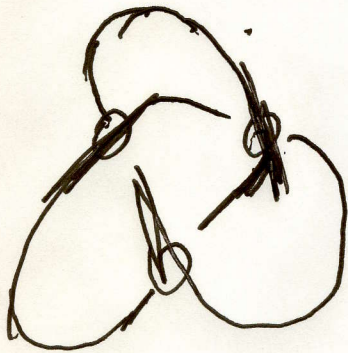
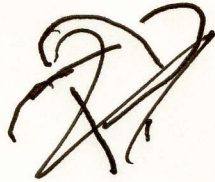
$$\varphi(\text{Knot with } >1 \text{ crossings}) = 0$$

$$\varphi(\text{Crossing}) - \varphi(\text{Crossing}) = 0$$

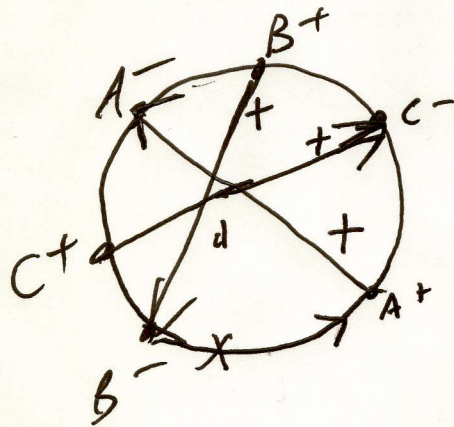
INVARIANTS OF ORDER 1 = constant f_n

$$g = e^h$$

GAUSS DIAGRAM OF A KNOT



$A^+ C^- B^+ A^- C^+ B^-$



Reidemeister moves

R1 $\infty =) = \infty$

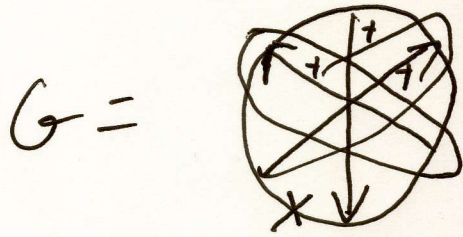
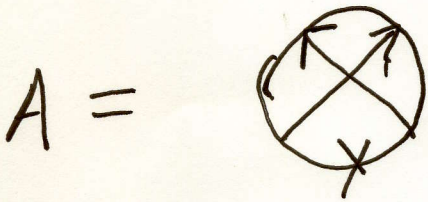
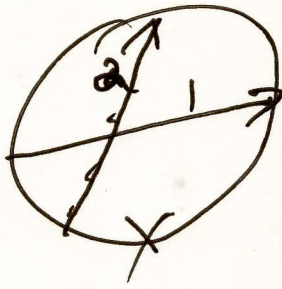
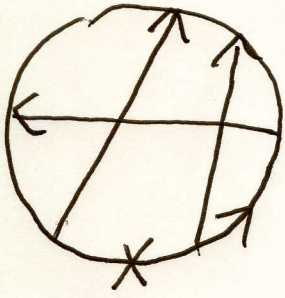
R2 $\begin{array}{c} \diagdown \\ \diagup \end{array} = \begin{array}{c} \diagup \\ \diagdown \end{array} = ||$

R3 $\begin{array}{c} \diagdown \\ \diagup \end{array} = \begin{array}{c} \diagup \\ \diagdown \end{array} \quad \begin{array}{c} \diagdown \\ \diagup \end{array} = \begin{array}{c} \diagup \\ \diagdown \end{array}$

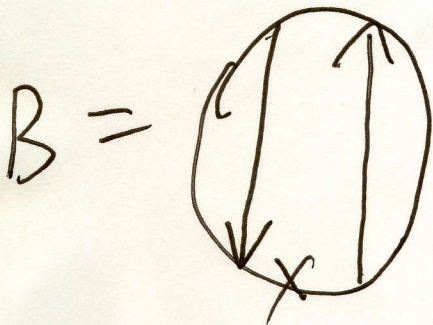
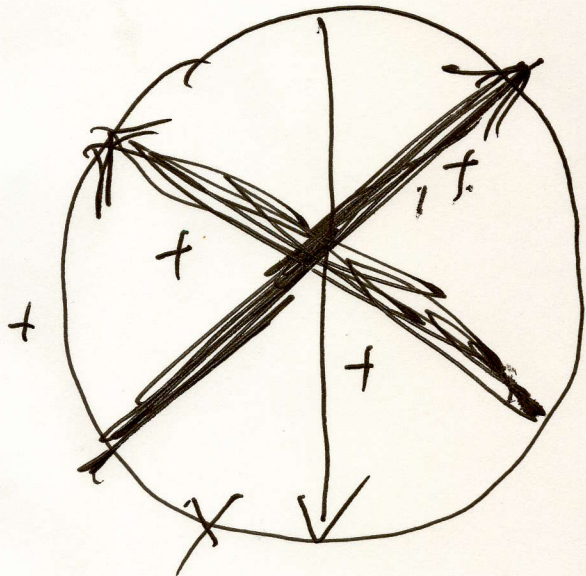
$\begin{array}{c} \text{---} \\ \curvearrowright \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \curvearrowleft \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \curvearrowright \\ \text{---} \end{array}$

$\begin{array}{c} \text{---} \\ \uparrow \uparrow \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \curvearrowright \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \uparrow \downarrow \\ \text{---} \end{array}$

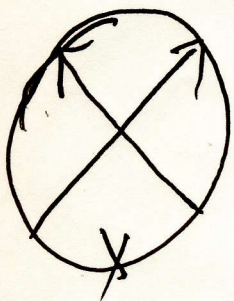
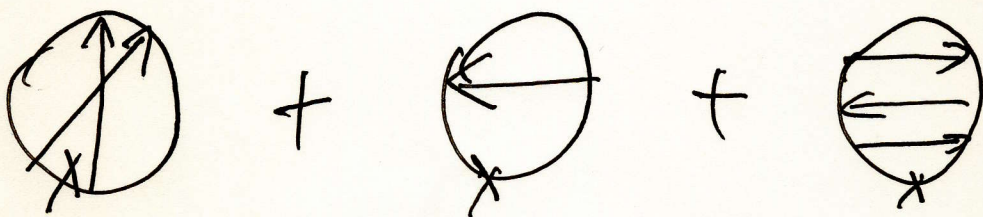
$\begin{array}{c} \uparrow \uparrow \\ \text{---} \\ \downarrow \downarrow \end{array} = \begin{array}{c} \uparrow \uparrow \\ \text{---} \\ \downarrow \downarrow \end{array} \quad \begin{array}{c} \uparrow \uparrow \\ \text{---} \\ \downarrow \downarrow \end{array} = \begin{array}{c} \uparrow \uparrow \\ \text{---} \\ \downarrow \downarrow \end{array}$



$$\langle A, G \rangle = 1$$



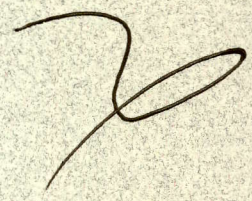
$$\langle B, G \rangle = 0$$



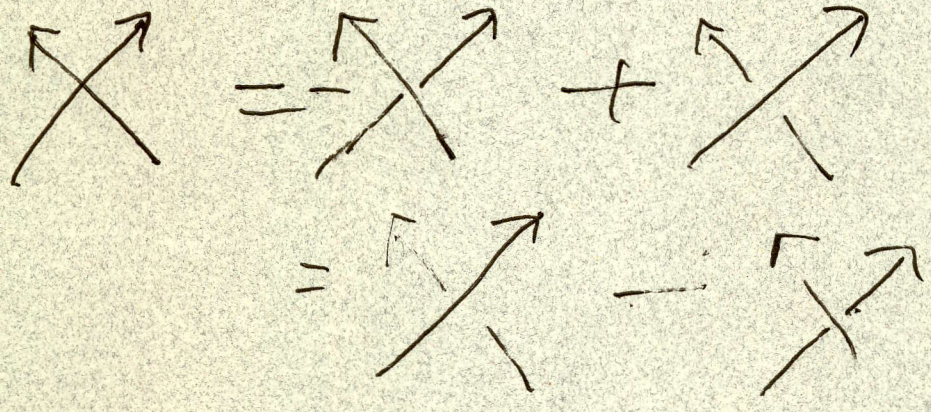
VASSILIEV INVARIANTS

GAUSS DIAGRAMS

ARROW DIAGRAMS (Product)



Goussarov proved that things:



$$\begin{aligned}
 \gamma_3(K) &= \langle -3 \text{ (diagram)} - 6 \text{ (diagram)} \\
 &= -6 \langle \text{ (diagram)} + \text{ (diagram)} \\
 &\quad + \text{ (diagram)} + \text{ (diagram)} \rangle
 \end{aligned}$$