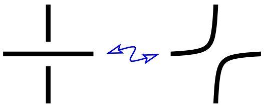
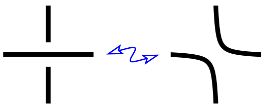
Knots & Links

The Kauffman bracket and the Jones polynomial [Ka1].

Let L be a link diagram.

A-splitting: 

B-splitting: 

A *state* S is a choice of either A- or B-splitting at every classical crossing.

$\alpha(S) = \#(\text{of } A\text{-splittings in } S)$

$\beta(S) = \#(\text{of } B\text{-splittings in } S)$

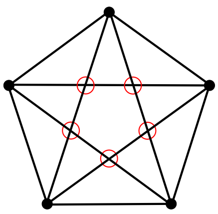
$\delta(S) = \#(\text{of circles in } S)$

$$[L](A, B, d) \ := \sum_S A^{\alpha(S)}\, B^{\beta(S)}\, d^{\delta(S)-1}$$

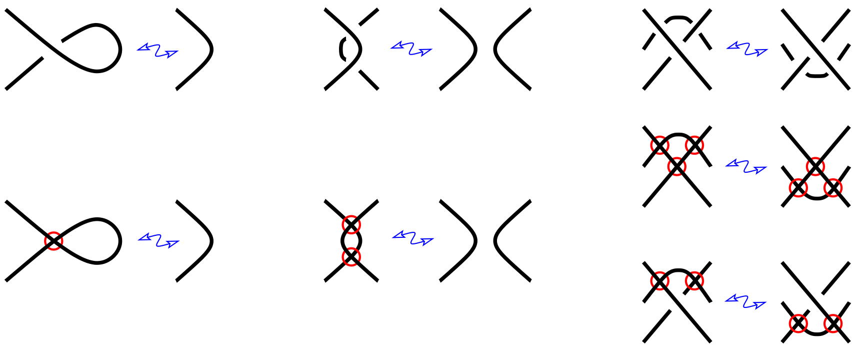
$$J_L(t) \ := (-1)^{w(L)} t^{3w(L)/4} [L](t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2})$$

Virtual links [Ka3].

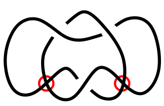
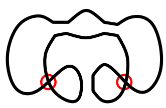
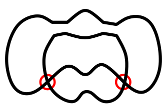
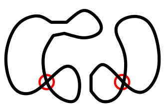
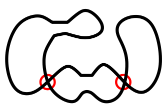
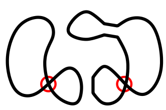
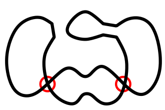
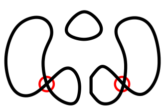
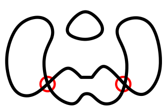
Virtual crossings



Reidemeister moves

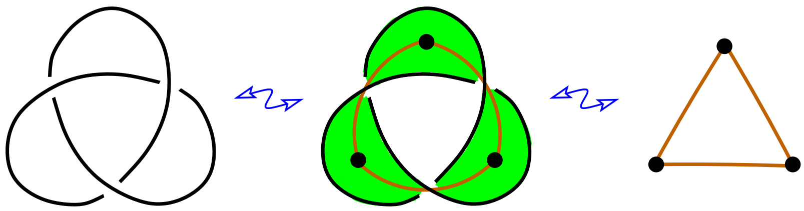


Example

				
(α, β, δ)	$(3, 0, 1)$	$(2, 1, 2)$	$(2, 1, 2)$	$(1, 2, 1)$
				
	$(2, 1, 2)$	$(1, 2, 1)$	$(1, 2, 3)$	$(0, 3, 2)$

$$[L] = A^3 + 3A^2Bd + 2AB^2 + AB^2d^2 + B^3d; \qquad J_L(t) = 1$$

Thistlethwaite's Theorem [Ka1] *Up to a sign and multiplication by a power of t the Jones polynomial $J_L(t)$ of an alternating link L is equal to the Tutte polynomial $T_\Gamma(-t, -t^{-1})$.*



The theorem was generalized to non-alternating links (using signed graphs in [Ka2] and using the Bollobás-Riordan polynomial for ribbon graphs in [DFKLS]; and virtual links in [ChVo].

Theorem [ChVo]. *Let L be an virtual link and G_L be the corresponding ribbon graph. Then*

$$[L](A, B, d) = A^n B^r d^{k-1} R_{G_L}\left(\frac{Ad}{B}, \frac{Bd}{A}, \frac{1}{d}\right) .$$

References

[ChVo] S. Chmutov, J. Voltz, *Thistlethwaite's theorem for virtual links*. Preprint `math.GT/0704.1310`.

[DFKLS] O. Dasbach, D. Futer, E. Kalfagianni, X.-S. Lin, N. Stoltzfus, *The Jones polynomial and dessins d'enfant*, Preprint `math.GT/0605571`.

[Ka1] L. H. Kauffman, *New invariants in knot theory*, Amer. Math. Monthly **95** (1988) 195–242.

[Ka2] L. H. Kauffman, *A Tutte polynomial for signed graphs*, Discrete Appl. Math. **25** (1989) 105–127.

[Ka3] L. Kauffman, *Virtual knot theory*, European Journal of Combinatorics, **20** (1999) 663–690.