

Knots and Graphs
Working Group [Summer 2009]
MATH 693, 3 credits
Instructor: *Sergei Chmutov*

RESEARCH PROJECTS

Project 1. *Virtual links.*

There are two different definitions of finite type virtual knot invariants due to L. Kauffman [Ka3] and M. Goussarov, M. Polyak, O. Viro [GPV]. The aim of this project is to compare these two definitions and to clarify the relations between them. We plan to start with the Jones polynomial and analyze the coefficients of its Taylor expansion after a certain substitution. We also plan to obtain new virtual knot invariants. References [Ka3, ChPo] contain further information about virtual links.

Project 2. *Rational knots.*

From any rational number (finite continued fraction) one may construct a knot [Con]. The goal is to relate the polynomial invariants (Jones, HOMFLY) of such rational knots with operations on continued fractions and with polynomial invariants of graphs (Tutte, Bollobás-Riordan). Some special cases of such relations were found in [J, Tr, Mof].

Project 3. *Matrix-tree theorem.*

A classical matrix-tree theorem expresses the determinant of some matrix constructed from a graph (principal minor of the Laplacian) as a sum over all spanning trees of the graph. We are going to study this theorem, its generalizations, and its application to knot theory. The main references here are [MV, BS, Ab].

Project 4. *Relative Tutte polynomial*

Recently a relative version of the Tutte polynomial of a graph with respect to a subset of its edges was discovered in [DH]. We are going to generalize various nice properties of the Tutte polynomial to its relative version. In particular we will look for a rank generating expression for the relative Tutte polynomial. For introduction to the Tutte polynomial see [EMM].

References

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