Definitions of matroids

References: J. Oxley [Ox], H. Whitney [Wh].

Independent sets.

A matroid is a pair $(E,\mathcal{I})$ consisting of a finite set E and a nonempty collection $\mathcal{I}$ of its subsets, called *independent sets*, satisfying the axioms:

(**I1**) *Any subset on an independent set is independent.*

(**I2**) *If X and Y are independent and $|X| = |Y| + 1$, then there is an element x in $X - Y$ such that $Y \cup \{x\}$ is independent.*

Circuits.

A matroid is a pair $(E,\mathcal{C})$ consisting of a finite set E and a nonempty collection $\mathcal{C}$ of its subsets, called *circuits*, satisfying the axioms:

(**C1**) *No proper subset of a circuit is a circuit.*

(**C2**) *If C_1 and C_2 are distinct circuits and $c \in C_1 \cap C_2$, then $(C_1 \cup C_2) - \{c\}$ contains a circuit.*

Bases.

A matroid is a pair $(E,\mathcal{B})$ consisting of a finite set E and a nonempty collection $\mathcal{B}$ of its subsets, called *bases*, satisfying the axioms:

(**B1**) *No proper subset of a base is a base.*

(**B2**) *If B_1 and B_2 are bases and $b_1 \in B_1 - B_2$, then there is an element $b_2 \in B_2 - B_1$ such that $(B_1 - \{b_1\}) \cup \{b_2\}$ is a base.*

Rank function.

A matroid is a pair (E,r) consisting of a finite set E and a function r, *rank*, assigning a number to a subset of E and satisfying the axioms:

(**R1**) *The rank of an empty subset is zero.*

(**R2**) *For any subset X and any element $y \notin X$,*

$$r(X \cup \{y\}) = \begin{cases} r(X)\ , & \textit{or} \\ r(X) + 1\ . & \end{cases}$$

(**R3**) *For any subset X and two elements y,z not in X, if $r(X \cup \{y\}) = r(X \cup \{z\}) = r(X)$, then $r(X \cup \{y,z\}) = r(X)$.*

Tutte polynomial.

$$\boxed{T_M(x,y) \ := \sum_{X \subseteq E} (x-1)^{r(E)-r(X)}(y-1)^{n(X)}}$$

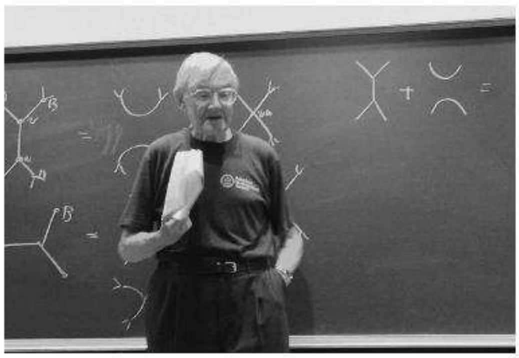

Fig. 1. W.T. Tutte. Photograph taken by Michel Las Vergnas at the CRM workshop, Barcelona, September 2001.

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References

[Ox] J. Oxley, *What is a matroid?*, preprint `http://www.math.lsu.edu/ oxley/survey4.pdf`.

[Wh] H. Whitney, *On the abstract properties of linear dpendence*, Amer. J. Math. **57**(3) (1935) 509–533.