Knots and Graphs

Working Group [Summer 2010]

MATH 693, class number 18761, 3 credits

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RESEARCH PROJECTS

Project 1. ***Matrix-tree type theorems in knot theory.***

A classical matrix-tree theorem expresses the determinant of some matrix constructed from a graph (principal minor of the Laplacian) as a sum over all spanning trees of the graph. This theorem has a generalization to hypergraphs or simplicial complexes to so called *Pfaffian matrix-tree theorem* [MV]. This theorem provides a formula for the first non-zero coefficient of the Conway polynomial of a link. We are going to generalize this theory to virtual links.

Project 2. ***Generalization of the relative Tutte polynomial***

A relative version of the Tutte polynomial of a graph with respect to a subset of its edges is a generalization of the ordinary Tutte polynomial. It was an important application in knot theory [DH]. The Jones polynomial of a virtual link is a specialization of it. However a generalization of the Jones polynomial, the arrow polynomial [DK] cannot be obtain as a specialization of the relative Tutte polynomial. We are going to generalize the relative Tutte polynomial so to cover the arrow polynomial.

Project 3. ***Polynomial invariants of graphs on surfaces***

There are three polynomial invariants of graphs embedded into surfaces. One is the Las Vergnas polynomial [LV1, LV2] coming from matroids. Another one is the Bollobás-Riordan polynomial [BR], a straightforward generalization of the Tutte polynomial. The third one is the Krushkal polynomial [Kr] defined using the symplectic structure in the first homology group of the surface. Each of them satisfies a contraction-deletion relation. According to the universality theorem [BR] any such polynomial can be expressed in terms of the Bollobás-Riordan polynomial. We propose to find these expressions explicitly and systematically investigate of the relations between these polynomials.

Project 4. ***Bollobás-Riordan polynomial and relative Tutte polynomial***

The classical theorem of M. Thistlethwaite relates the Jones polynomial to the Tutte polynomial of some special graph. There are different two generalizations of this theorem to virtual links. One [Ch] uses the Bollobás-Riordan polynomial. Another [DH] uses the relative Tutte polynomial. The comparison of these results indicated that perhaps there is a relations between these two polynomials. We are going to try to find it.

References

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