

Li Hoon Chun



$$= \text{Diagram 1} + \text{Diagram 2}$$

Diagram 1: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 2: A vertex labeled 1 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$= \left(\text{Diagram 3} + \text{Diagram 4} \right) + \left(\text{Diagram 5} + \text{Diagram 6} \right)$$

Diagram 3: Two vertices labeled 2 are connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 4: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 5: A vertex labeled 1 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 6: A vertex labeled 1 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$= \left(\left[\text{Diagram 7} + \text{Diagram 8} \right] + \left[\text{Diagram 9} + \chi_1 \text{Diagram 10} \right] \right)$$

Diagram 7: A vertex labeled 3 is connected to two vertices labeled 2, which are then connected to each other by an edge.

Diagram 8: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 9: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 10: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$+ \left(\left[\text{Diagram 11} + \chi_1 \text{Diagram 12} \right] + \left[\text{Diagram 13} + \chi_1 \text{Diagram 14} \right] \right)$$

Diagram 11: A vertex labeled 1 is connected to two vertices labeled 2, which are then connected to each other by an edge.

Diagram 12: A vertex labeled 1 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 13: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 14: A vertex labeled 1 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$= \left(\left[y \text{Diagram 15} + \left(\text{Diagram 16} + \chi_2 \text{Diagram 17} \right) \right] + \left[\left(\text{Diagram 18} + \chi_1 \text{Diagram 19} \right) + \right. \right.$$

Diagram 15: A vertex labeled 3 is connected to two vertices labeled 2, which are then connected to each other by an edge.

Diagram 16: A vertex labeled 3 is connected to two vertices labeled 2, which are then connected to each other by an edge.

Diagram 17: A vertex labeled 1 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 18: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 19: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$\left. \chi_1 \left(\text{Diagram 20} + \chi_1 \text{Diagram 21} \right) \right] + \left(\left[\left(\text{Diagram 22} + \chi_1 \text{Diagram 23} \right) + \chi_1 \left(\text{Diagram 24} \right. \right. \right.$$

Diagram 20: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 21: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 22: A vertex labeled 3 is connected to two vertices labeled 2, which are then connected to each other by an edge.

Diagram 23: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 24: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$+ \chi_1 \text{Diagram 25} \left. \right] + \left[\left(\text{Diagram 26} + \chi_1 \text{Diagram 27} \right) + \chi_1 \left(\text{Diagram 28} + \right. \right.$$

Diagram 25: A vertex labeled 1 is connected to two vertices labeled 2, which are then connected to each other by an edge.

Diagram 26: A vertex labeled 3 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 27: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

Diagram 28: A vertex labeled 2 is connected to two vertices labeled 1, which are then connected to each other by an edge.

$$x_1 \langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle \rangle \rangle$$

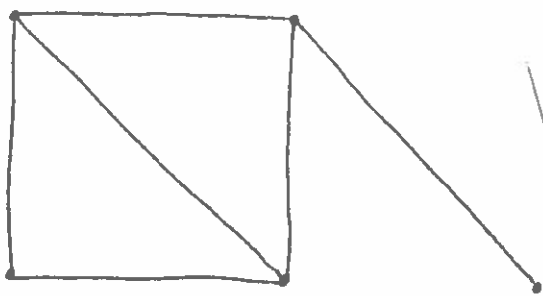
$$= ([y(yx_5 + x_5 + x_3x_2) + ([yx_5 + x_5 + x_3x_2] + x_2[yx_3 + x_3 + x_1x_2])]) + ([([yx_5 + x_5 + x_2x_3] + x_1[yx_4 + x_4 + x_2^2]) + x_1([yx_4 + x_4 + x_2^2] + x_1[yx_3 + x_3 + x_2x_1])]) + ([([yx_5 + x_5 + x_3x_2] + x_1[yx_4 + x_4 + x_2^2]) + x_1([yx_4 + x_4 + x_2^2] + x_1[yx_3 + x_3 + x_1x_2])]) + ([([x_5 + x_1x_4 + x_1x_4 + x_1^2x_3] + x_1[x_4 + x_1x_3 + x_1x_3 + x_1^2x_2]) + x_1([x_4 + x_1x_3 + x_1x_3 + x_1^2x_2] + x_1[x_3 + x_1x_2 + x_1x_2 + x_1^3])])]$$

$$= \underline{y^2x_5} + \underline{yx_5} + \underline{yx_3x_2} + \underline{yx_5 + x_5} + \underline{x_3x_2} + \underline{yx_2x_3} + \underline{x_2x_2} + \underline{x_1x_2} + \underline{yx_5 + x_5 + x_2x_3} + \underline{yx_1x_4} + \underline{x_1x_4} + \underline{x_1x_2} + \underline{yx_1x_4} + \underline{x_1x_4} + \underline{x_1x_2^2} + \underline{yx_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{yx_5 + x_5 + x_3x_2} + \underline{yx_1x_4} + \underline{x_1x_4} + \underline{x_1x_2^2} + \underline{yx_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_5} + \underline{x_1x_4} + \underline{x_1x_4} + \underline{x_1^2x_3} + \underline{x_1x_4} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_1x_4} + \underline{x_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_1^5}$$

$$= y^2(x_5) + y(4x_5 + 4x_4x_1 + 2x_3x_2 + 2x_1^2x_3) + (4x_5 + 8x_4x_1 + 4x_3x_2 + 5x_1x_2^2 + 8x_1^2x_3 + 6x_1^3x_2 + x_1^5)$$

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$$\begin{aligned}
 a \circ b &= \bigcirc^{a+b} + a \text{---} b \\
 &= y \chi_{a+b} + (\bigcirc^{a+b} + a \text{---} b) \\
 &= y \chi_{a+b} + \chi_{a+b} + \chi_a \chi_b \\
 &= \bigcirc^{a+b+c} + \chi_a (\bigcirc^{b+c}) \\
 &= (\bigcirc^{a+b+c} + \bigcirc^{a+b} \bigcirc^c) + \chi_a (\bigcirc^{b+c} + \bigcirc^b \bigcirc^c) \\
 &= \chi_{a+b+c} + \chi_a \chi_{b+c} + \chi_{a+b} \chi_c \\
 &\quad + \chi_a \chi_b \chi_c
 \end{aligned}$$



$$= \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} + \chi_1 \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array}$$

$$= \left(\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 + \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \right) + \chi_1 \left(\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 + \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \right)$$

$$= \left(\left[\begin{array}{|c|c|} \hline 3 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 + \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 \right] + \left[\begin{array}{|c|c|} \hline 2 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 + \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 \right] \right)$$

$$+ \chi_1 \left(\left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 + \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 \right] + \left[\begin{array}{|c|c|} \hline 2 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 + \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 \right] \right)$$

$$= \left(\left[y \begin{array}{|c|c|} \hline 3 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 + \left(\begin{array}{|c|c|} \hline 3 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 + \chi_1 \begin{array}{|c|c|} \hline 2 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 \right) \right] + \left[\left(\begin{array}{|c|c|} \hline 2 & 3 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^3 + \begin{array}{|c|c|} \hline 2 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 \right) \right. \right.$$

$$\left. + \left(\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 2 & 1 \\ \hline \end{array} \bigcirc^2 + \chi_1 \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 \right) \right] + \chi_1 \left(\left[y \begin{array}{|c|c|} \hline 3 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 + \left(\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 + \right. \right.$$

$$\left. \chi_1 \begin{array}{|c|c|} \hline 2 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 \right) \right] + \left[\left(\begin{array}{|c|c|} \hline 2 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^2 + \begin{array}{|c|c|} \hline 2 & 1 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 \right) + \left(\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 2 & 1 \\ \hline \end{array} \bigcirc^1 + \chi_1 \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \hline 1 & 1 \\ \hline \end{array} \bigcirc^1 \right) \right]$$

$$\begin{aligned}
&= \left(\left[y(yx_5 + x_5 + x_2x_3) \right] + \left(\left[yx_5 + x_5 + x_2x_3 \right] + \right. \right. \\
&x_1 \left[yx_4 + x_4 + x_2^2 \right] \left. \right) \left. \right] + \left(\left[yx_5 + x_5 + x_3x_2 \right] + \right. \\
&\left[x_5 + x_2x_3 + x_4x_1 + x_1x_2^2 \right] \left. \right) + \left(\left[x_5 + x_3x_2 + x_1x_4 + x_1x_2^2 \right] \right. \\
&+ x_1 \left[x_4 + x_1x_3 + x_3x_1 + x_1^2x_2 \right] \left. \right) \left. \right) + x_1 \left(\left[y(yx_4 + x_4 + \right. \right. \\
&x_3x_1) + \left(\left[yx_4 + x_4 + x_3x_1 \right] + x_1 \left[yx_3 + x_3 + x_1x_2 \right] \right) \left. \right] + \\
&\left[\left(\left[yx_4 + x_4 + x_2^2 \right] + \left[x_4 + x_1x_3 + x_3x_1 + x_1^2x_2 \right] \right) + \left(\left[x_4 + \right. \right. \right. \\
&x_1x_3 + x_2^2 + x_1^2x_2 \left. \right] + x_1 \left[x_3 + x_1x_2 + x_2x_1 + x_1^3 \right] \left. \right) \left. \right] \left. \right)
\end{aligned}$$

$$\begin{aligned}
&= \underline{y^2x_5} + \underline{yx_5} + \underline{yx_2x_3} + \underline{yx_5} + \underline{x_5} + \underline{x_2x_3} + \underline{yx_1x_4} + \underline{x_1x_4} + \\
&\underline{x_1x_2^2} + \underline{yx_5} + \underline{x_5} + \underline{x_3x_2} + \underline{x_5} + \underline{x_2x_3} + \underline{x_4x_1} + \underline{x_1x_2^2} + \underline{x_5} \\
&+ \underline{x_3x_2} + \underline{x_1x_4} + \underline{x_1x_2^2} + \underline{x_1x_4} + \underline{x_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \\
&\underline{y^2x_1x_4} + \underline{yx_1x_4} + \underline{yx_1^2x_3} + \underline{yx_1x_4} + \underline{x_1x_4} + \underline{x_1^2x_3} + \\
&\underline{yx_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{yx_1x_4} + \underline{x_1x_4} + \underline{x_1x_2^2} + \underline{x_1x_4} + \\
&\underline{x_1^2x_3} + \underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_1x_4} + \underline{x_1^2x_3} + \underline{x_1x_2^2} + \underline{x_1^3x_2} + \\
&\underline{x_1^2x_3} + \underline{x_1^3x_2} + \underline{x_1^3x_2} + \underline{x_1^5}
\end{aligned}$$

$$\begin{aligned}
&= y^2(x_5 + x_1x_4) \\
&+ y(3x_5 + 4x_4x_1 + x_3x_2 + 2x_1^2x_3) \\
&+ (4x_5 + 8x_4x_1 + 4x_3x_2 + 5x_1x_2^2 + 8x_1^2x_3 + 6x_1^3x_2 + \\
&x_1^5).
\end{aligned}$$

THEY ARE EQUAL if $y=0$.